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Virtual Physics

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Exercise 2: Doing it the hard way

This exercise may take some time and is not easy

In this exercise, we shall create and simulate a mathematical model in the old-fashioned, traditional way. Hence an old-fashioned, traditional example seems appropriate: let us go to the circus. At the end of the show, the circus presenter announces the highlight of the evening: a motorcyclist riding on the high wire 15 meters above the ground. Of course, there is an extra level of difficulty. Not only the artist needs to balance the motorcycle, also he is required to balance a girl on a swing!

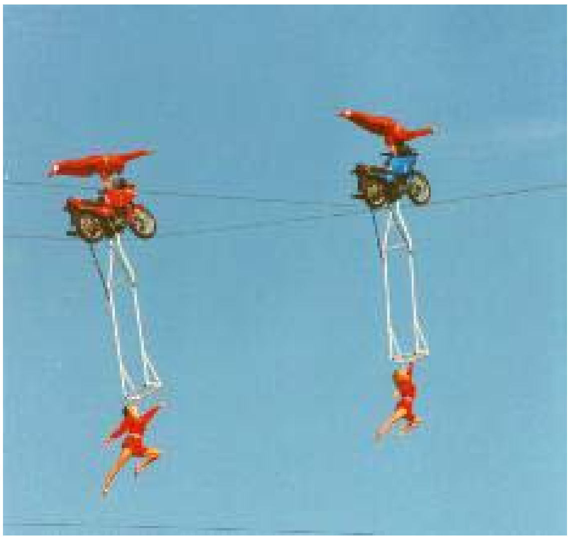

Figure 1: Two motorcyclists and two pretty ladies

For any person with a proper education in physics, this last announcement will take away all the excitement. By attaching a swing to motorcycle, the center of gravity of the combined device is now lower than the wire and the motorcycle actually hangs safely in the height. In this way, a tricky balancing act is reduced to a physical triviality. All what is needed by the circus artists is pretentious acting that makes the ride appear more dangerous than it is.

Whereas the balancing has lost most of its thrill, the influence of the swinging on the movement of the motorcycle still remains interesting. How will the swinging motion accelerate or decelerate the motorcycle? For this analysis, we need a suitable physical model of the motorcycle. By simulating it, we can then determine the motion of the system.

Stage 1: Model Abstraction

Any modeling effort is based on assumptions. By stating these modeling assumptions, we define a suitable level of abstraction:

- We assume that the balancing is irrelevant. Only the vertical dimension and the dimension along the wire are of interest. The three-dimensional space is thereby reduced to a two-dimensional plane.
- We assume the wire to be so tightened that it can be regarded as rigid with the shape of a straight line.

- We assume all other mechanical elements to be rigid and frictionless.
- We assume both the rider with his motorcycle as well as the girl in its swing to be represented by point masses.

These assumptions enable us to abstract the device, with all its complex geometries, elasticity and friction coefficients to a very simple system of two rigid bodies on a single plane.

Stage 2: Model Decomposition

The motorcycle with swing can now be regarded as a sliding block with a pendulum attached to it. Figure 2 depicts this abstraction and displays the decomposition of our model in two separate components that each represent a rigid body.

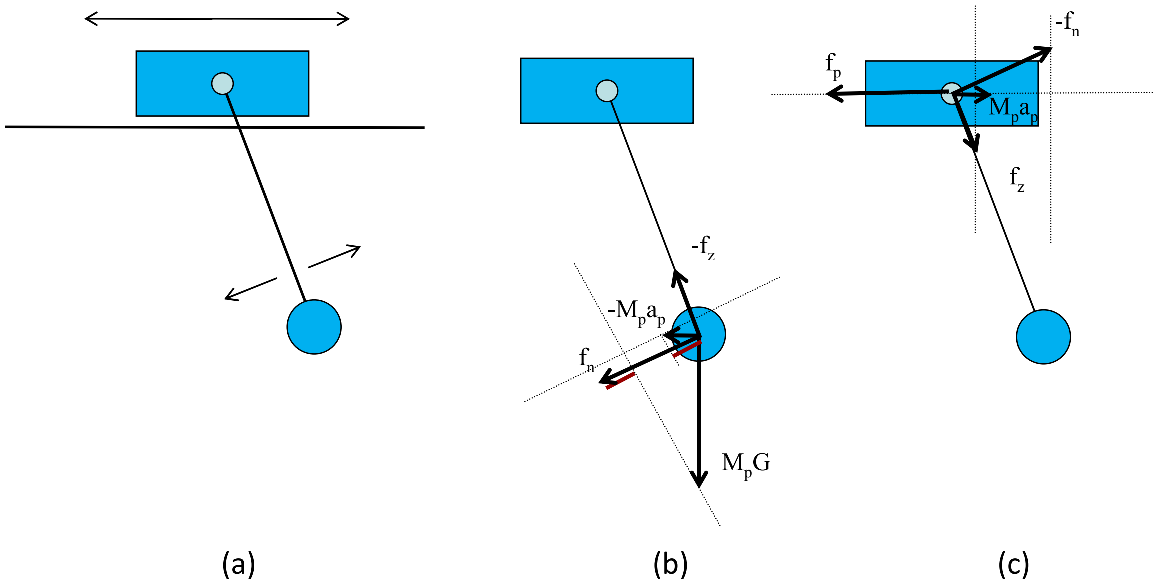

Figure 2: **(a)** Interpreation of the motorcyclist with swing as two-body system **(b)** Forces determining the normal force **(c)** Force balance at the hinge point

Stage 3: Component Equations

The sliding body and the pendulum form components of our system. Let us derive the equations for both of these components separately. We start with the sliding body. It is constrained in its movement to the horizontal domain and driven by the force f_S. This stated by Newton's law:

$$f_S = M_S \cdot a_S$$

The actual movement is then described by two differential equations for translational velocity v and acceleration a_S. Velocity is defined as the time-differential of position s, and acceleration is the time-differential of velocity.

$$a_S = dv/dt$$
$$v = ds/dt$$

These three simply equations are all what we need to describe the motion of the sliding body.

The motion of the pendulum that represents the swing is best described by rotational quantities such as angle φ, angular velocity ω, and angular acceleration α. The corresponding equations strongly resemble their counterparts in the translational domain.

$$\alpha = d\omega/dt$$
$$\omega = d\varphi/dt$$

The pendulum is constrained in its movement to the rotational domain. In this domain, the corresponding counterpart to Newton's law is stated as:

$$\tau = I \cdot \alpha$$

Presuming that M_P is a point mass, the corresponding rotational inertia is represented by a constant value $I := M_P * R^2$. The torque t is the rotational counterpart to force and is defined as the product of the radius and the force f_n that is normal to the radius.

$$\tau = f_n \cdot R$$

In contrast to the sliding body, not all forces act on the center of mass. Hence the balance of forces is significantly more complicated for the pendulum. The normal force consists of two components that both originate from an external acceleration of the mass. The vertical force component is $M_P \cdot G$. Its cause is the gravity acceleration $G := -9.81 m/s^2$. The horizontal component is $-M_P \cdot a_P$. It is originating from the horizontal acceleration of a_P of the hinge point. Using the trigonometric functions sinus and cosinus, we can project both components on the normal direction in dependence on the angle φ.

$$f_n = \sin(\varphi) \cdot M_P \cdot G - \cos(\varphi) \cdot M_P \cdot a_P$$

Finally, we have to compute the horizontal force f_P acting on the hinge point. This force must be in balance with three other foces. The first one is the horizontal component from the centripetal force f_z. The centripetal force in direction of the revolution center and is required to keep the mass on a circular path. It is defined to be

$$f_z = M_P \cdot R \cdot \omega^2$$

The second force is the horizontal component of the inverted normal force $-f_{n..}$. The third force represnts the reaction force to the imposed horizontal acceleration a_P of the mass M_P. Again this is computed by the term: $M_P \cdot a_P$

D'Alembert's Principle states that the reaction force of an imposed motion must cancel out the acting forces. Using this principle, we derive the equation:

$$f_P + \sin(\varphi) \cdot f_z - \cos(\varphi) \cdot f_n - M_P \cdot a_P = 0$$

This is the last equation of the pendulum.

Stage 4: Total System Equations

We now have a set of equations for both bodies: the sliding body is described by 4 variables and 3 equations; the pendulum is described by 8 variables and 7 equations. In order to get a structurally regular system, the number of equations must equal the number of variables. Hence we are missing two equations. These result from the connection of the two bodies.

The acceleration at the hinge points must be equal:

$$a_P = a_S$$

The two horizontal forces must cancel out at the hinge point:

$$f_P + f_S = 0$$

Finally, our model is described by a set of 12 variables and 12 equations:

$a = dv/dt$
$v = ds/dt$
$f_S = M_S \cdot a_S$
$\alpha = d\omega/dt$
$\omega = d\varphi/dt$
$\tau = I \cdot \alpha$
$\tau = f_n \cdot R$
$f_n = M_P \cdot (\sin(\varphi) \cdot G - \cos(\varphi) \cdot a_P)$
$f_z = M_P \cdot R \cdot \omega^2$
$f_P + \sin(\varphi) \cdot f_z - \cos(\varphi) \cdot f_n - M_P \cdot a_P = 0$
$a_P = a_S$
$f_P + f_S = 0$

with the constant parameters M_S:= 250kg, M_P := 70kg, R := 2.5m, and I := $M_P \cdot R^2$

Task 1: Punch in the equations into Modelica and simulate the model using Dymola

If you set up the initial velocities to zero and the initial angle to 1.25 rad, this is the simulation result you should get for 5 seconds of simulation time.

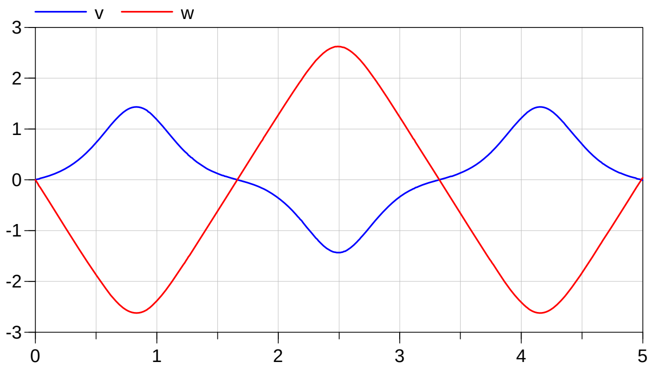

Figure 3: Translational and angular velocity over time [s]

Task 2: Generate the simulation code by yourself.

Derive the state space-form of the system. The state vector is described by **x** := (s, v, φ, ω). Transform the equation system into the form:

dx/dt := f(x);

Follow the scheme presented in Lecture 1B. Apply Forward Euler with a fixed step-size of 1ms and create a simulation code in Python.

Hint: You can assume all four state-variables to be known. From this, determine as many other variables as possible. For the remaining system of equations, substitute away all forces and torques until you end up with two equations for a_p *and* α*. You may solve for any of these two variables and determine the rest of the system by back-substitution.*
Be careful! Once you do a mistake, it will be hard to locate!