

# Virtual Physics

28.10.2014

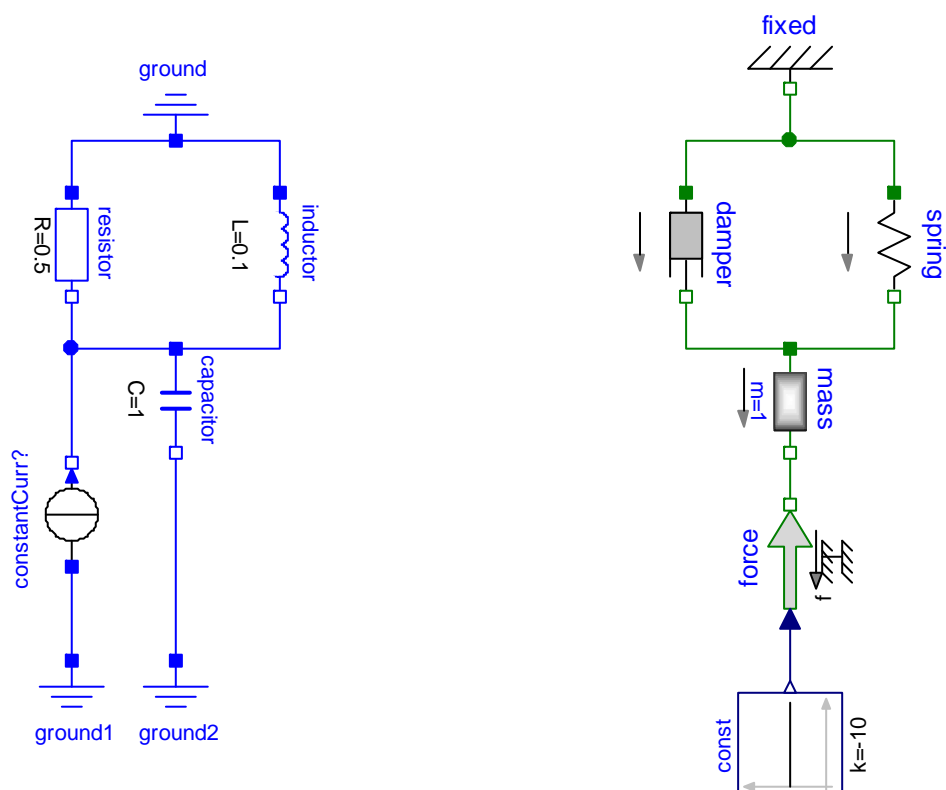
## Exercise 3: Mechanics, Electrics, Hydraulics

### Solution

**Task A: Model the equivalent electric circuit.**

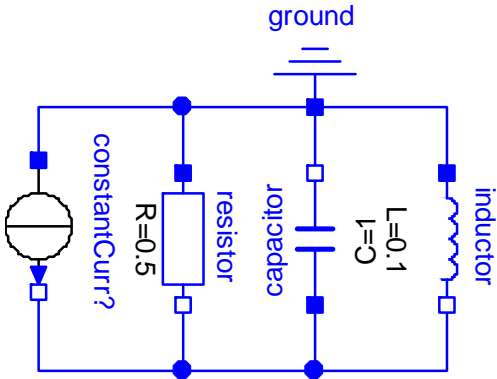
From the lecture slides we know the following analogs:

- Damper  $\rightarrow$  Resistor (with inverted parameter)
- Spring  $\rightarrow$  Inertia (with inverted parameter)
- Mass  $\rightarrow$  Capacitor
- Source of Force  $\rightarrow$  Source of Current
- Fixation  $\rightarrow$  Ground

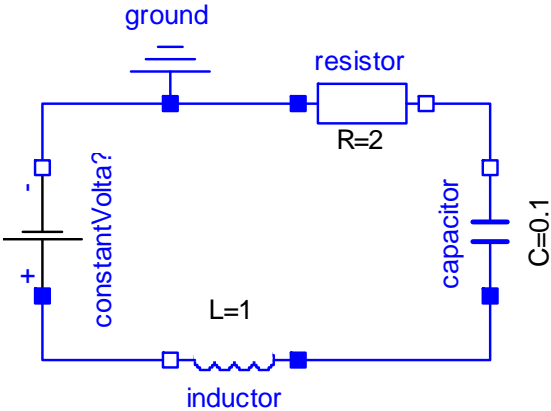


This pretty much prescribes the analog electric circuit. The only difference is the different design of mass and capacitor. The mass is implicitly “grounded” The capacitance is not. The mass is designed as two port element but just for the sake of convenience. The two ports express the same pair of variables, there is no difference of the potential variable across the mass element.

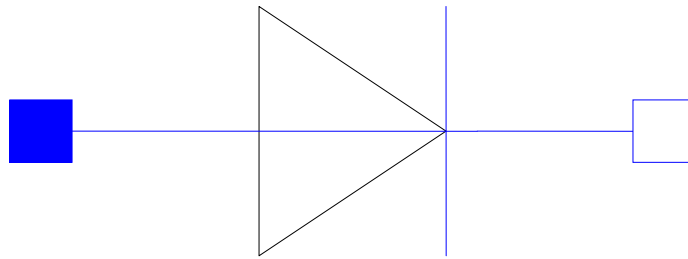
The individual grounds can be merged and the circuit can be further simplified.



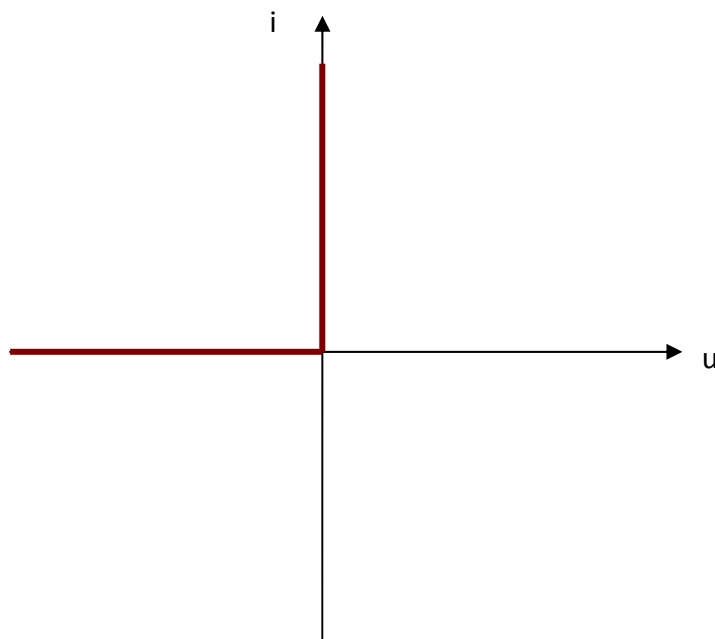
All components of this circuit share the same difference in voltage potential. There is also the dual circuit, where all components share the same current that expresses the velocity of the mass.



**Task B: An ideal diode.**



The characteristic curve of an ideal diode shares two semi-axes of the coordinate system.



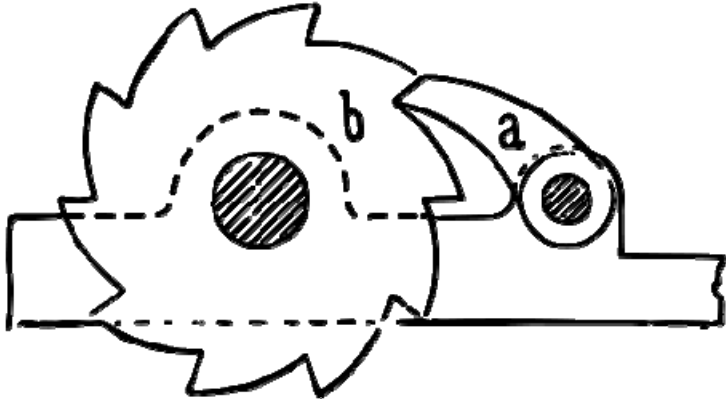
Such a curve can be described by a curve parameter  $s$ .

if  $s < 0$ :  $s = u$ ;  $i = 0$ ;


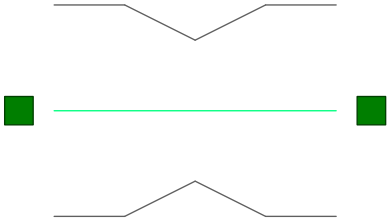
if  $s > 0$ :  $s = i$ ;  $u = 0$ ;

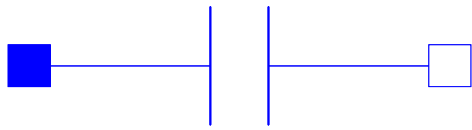
The product of  $u \cdot i$  will always be zero. So this component does not consume work and is energy neutral.

The mechanical equivalent to an ideal diode is a limited joint that moves only in one direction and stalls in the opposite direction. A ratchet-wheel used to coil up the cable of a vacuum-cleaner is one suitable example within rotational dynamics.



**Task C: Modeling Hydraulic Components**

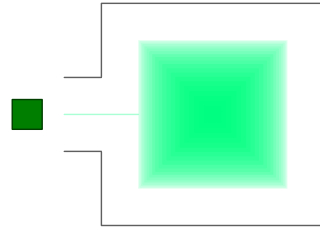
Electrics	Hydraulics
 <p>Resistor with Resistance <math>R</math>[Ohm = V/A]</p> $u = R \cdot i$ <p>A resistor dissipates electrical energy into heat.</p>	 <p>Pipe with linear Pressure drop due to laminar flow with <math>k</math> [Pa s/m<sup>3</sup>]</p> $\Delta p = k \cdot v$ <p>Energy is dissipated as heat.</p> <p><math>k</math> hereby results out of the Hagen–Poiseuille equation: <math>k = (L \cdot 8 \cdot \eta) / (\pi \cdot R^4)</math></p> <p>(L: pipe length, R: pipe radius, <math>\eta</math>: viscosity)</p>



Capacitor with capacitance  $C$  [ $F = C/V$ ]

$$du/dt \cdot C = I$$

A capacitor stores energy in an electric field built up by the charge of the capacitor



Compressible volume (only one inlet) with  $\kappa$  [ $\text{Pa}^{-1}$ ] as compressibility and  $V$  [ $\text{m}^3$ ] as volume

$$dp/dt \cdot \kappa \cdot V = v$$

Energy is stored by building up pressure in the volume.

### Electrics

### Hydraulics



Coil with inductance  $L$  [ $H = \text{Vs}^2/C$ ]

$$di/dt \cdot L = u$$

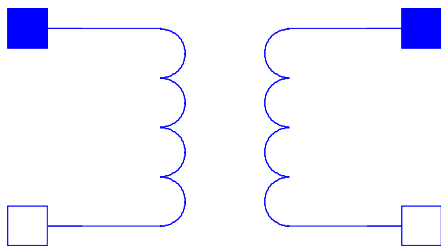
A coil stores energy in the magnetic field.



Hydraulic inductance by the acceleration of mass in a pipe with  $\rho$  [ $\text{kg}/\text{m}^3$ ] as volumetric density,  $A$  [ $\text{m}^2$ ] as area of inlet and outlet, and  $l$  [ $\text{m}$ ] as length of the pipe.

$$dv/dt \cdot \rho \cdot l/A = \Delta p$$

Energy is stored in form of kinetic energy. Dynamic pressure is built up.



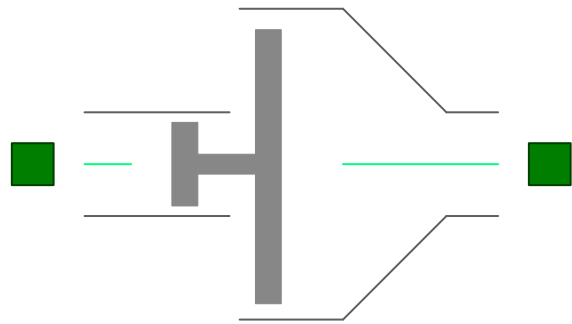
**M**

Electrical transformer with gain M [1]

$$u_2 = M \cdot u_1$$

$$M \cdot i_2 = i_1$$

It transforms energy and is non-dissipative.  
The transformation is done by a magnetic field in between. Effects due to resistance and inductivity are neglected.



Hydraulic transformer with gain M[1]

$$p_2 = M \cdot p_1$$

$$M \cdot v_2 = v_1$$

The transformation is done by a mechanic double piston. Effects due to resistance and inductivity are neglected.