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Virtual Physics

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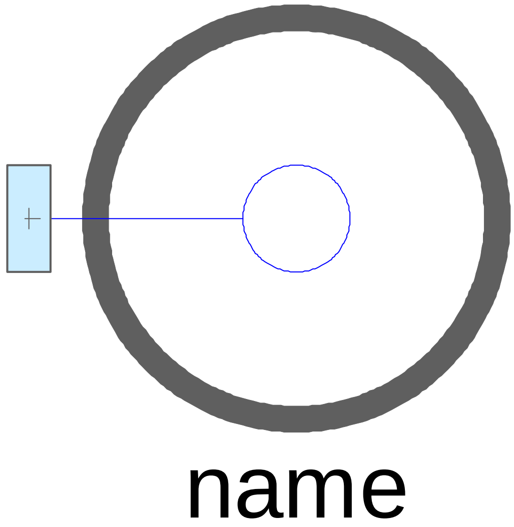
Exercise 7: 2D-Mechanics: Ideal rolling wheel

Task A: Develop a component for an ideal wheel joint.



Model a wheel joint (i.e. a wheel without mass and inertia) that exhibits an ideal rolling on the x-axis (or on the xz-plane in the 3D Visualization). Although, the wheel is regarded as joint-element, it has only one connector since the other connection point is implicitly represented by the x-axis.

Such an ideal rolling wheel imposes one holomic constraint for the position on the y-axis and one non-holonomic constraint in order to enforce ideal rolling.

$$y = R$$

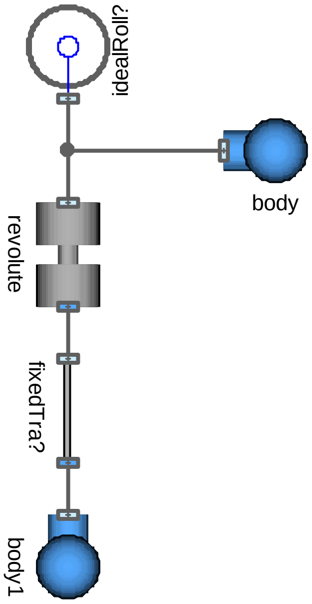
$$R \cdot \omega = v_x$$

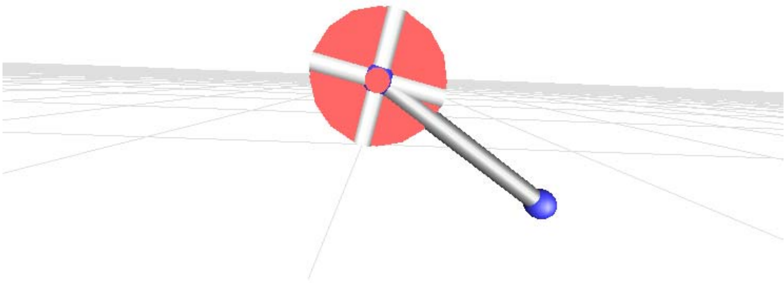
The wheel joint defines one parameter and that is the radius of the wheel. Since this rolling is restricted to a single dimension, the non-holonomic constraint may be elevated to a holonomic constraint.

Provide also a suitable 3D animation for your component.

Task B: Test your component

You can test your ideal wheel joint by using it as a replacement for the prismatic joint in the crane crap.





Task C (optional): Model a rigid wheel with dry friction

Instead of imposing a non-holonomic constraint equation, one may apply the law of dry friction. Implement a second wheel joint with a dry friction model and use it to remodel the ball with counter-spin of Lecture 5a. *Beware: the normal force is not a parameter of the model anymore. It results out of the holonomic constraint. Can the normal force become negative?*

Task D: Apply Pantelides Algorithm (taken from Exam WS 2010/2011)

Transform the following system of differential-algebraic system into state-space form.

```
dx/dt = 5 * z * b

dy/dt = a

2 * dz/dt = b

b = y * x

y = 1 – x

a = c – d

d/2 = b
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Causalize each equation and transform the set of equations into a sequence of assignments. You may differentiate equations if necessary.