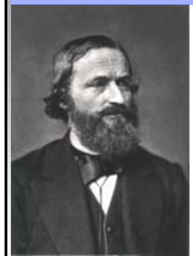
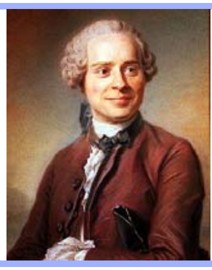
Virtual Physics
Equation-Based Modeling

TUM, October 14, 2014

Object-oriented formulation of physical systems – Part II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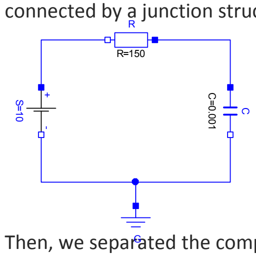
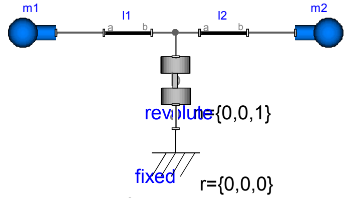
Common Modeling Approach

Attentive students may have noticed that we have done the same thing twice in the last hour.

- For mechanic or electric systems, the procedure was actually the same.
- First we decomposed the system into different components that are connected by a junction structure.





- Then, we separated the component equations from the connection equations.

Connector variables

- For each node in the junction structure, we defined a set of equations.
- Each node was represented by a pair of variables

  A **potential** variable

    v (voltage potential for electrics)

    v (velocity for mechanics)

  and a **flow** variable

    i (current for electrics)

    f (force for mechanics)

Connector equations

- For one connection between a set of n nodes, n equations have to be generated.

- **n-1 equalities**

  In electrics: $v_1 = v_2 = \dots = v_n$ (Kirchhoff`s 2nd law)

  In mechanics: $v_1 = v_2 = \dots = v_n$ (Rigid constraint equation)

- **1 balance equation**

  In electrics: $i_1 + i_2 + \dots + i_n = 0$ (Kirchhoff`s 1st law)

  In mechanics: $f_1 + f_2 + \dots + f_n = 0$ (D`Alembert`s Principle)

Energy flows

But there is more to it:

- What does the product of the **mechanic** pair of connector variables represent?

  v [m/s] · f [N] = p [Nm/s]

  It represents a flow of energy! [Nm] is work/energy

- What does the product of the **electric** pair of connector variables represent?

  v [Nm/C] · I [C/s] = p [Nm/s]

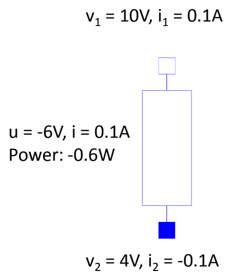
  It represents a flow of energy too!

  This is not a coincidence! It indicates a general physical principle!

Energy flows and power

Each component exhibits a certain behavior w.r.t. energy

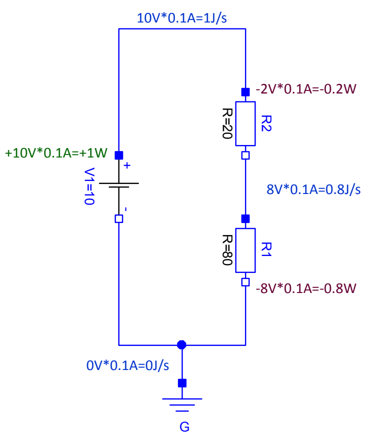
- There is a flow into the component at a certain level of energy.
- There is a flow out of the component at a possibly different level of energy.
- The difference between the two levels of energy represents **work**!
- The difference between the two flows represents **power!** (work per time)
- Energy is a potential size, whereas work represents the difference. This is the same distinction as between voltage and voltage potential.



Energy flows and power

Example for clarification

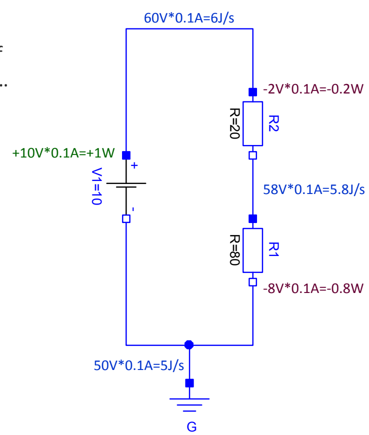
- If I change the grounding voltage of an electric circuit…



Energy flows and power

Example for clarification

- If I change the grounding voltage of an electric circuit (from 0V to 50V)…

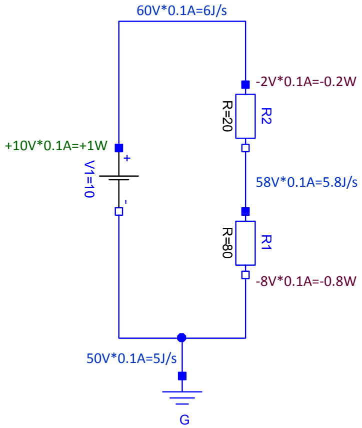


Energy flows and power

Example for clarification

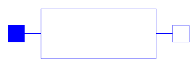
- If I change the grounding voltage of an electric circuit (from 0V to 50V)…
- …all energy flows at the connector change.
- But the power across the components remains the same!
- Potential variables are auxiliary variables. For the physical behavior, only the difference between potentials does matter.

(There are exceptions where the potentials cannot be chosen arbitrarily)



Energetic behavior

Some components dissipate energy



Resistor

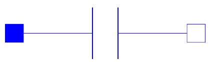
u = R*i

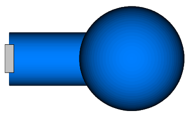
Damper

$\Delta v = D^{-1} * f$

Energetic behavior

Some components store energy (by integrating the flow variable):





Capacitor

du/dt · C = I

(Storage of charge)

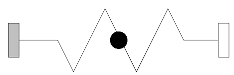
Mass

dv/dt · M = *f (Newton's Law)*

(Storage of kinetic energy)

Energetic behavior

Some components store energy (by integrating the potential variable):



Inductance

di/dt · L = *u*

(The energy is stored in the magnetic field)

Spring

$df/dt \cdot C^{-1} = \Delta v$

(Velocity is integrated to position)

(this is not a good analogy, though)

Energetic behavior

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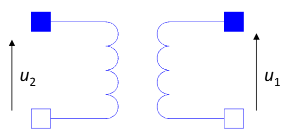
Some components transform energy

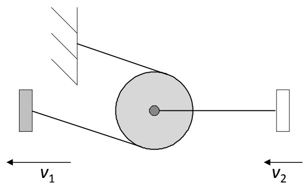


M

Transformer

$u_2 = M \cdot u_1$

$M \cdot i_2 = i_1$



Linkage

$v_2 = G \cdot v_1$

$G \cdot f_2 = f_1$

Energetic behavior

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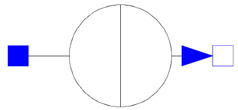
Some components represent a source or sink of energy



Current Source

i = I0

Constant force

f = f0

Energetic behavior

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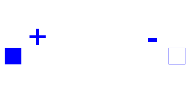
Some components represent a source or sink of energy



Voltage Source

u = U0

Constant velocity

Δv = V0

Sink or Source?

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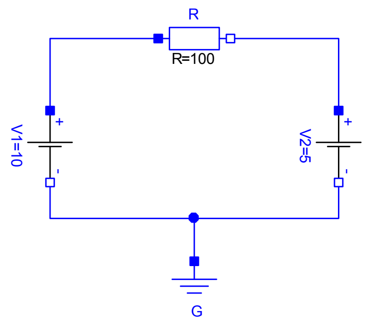

Sink or Source?

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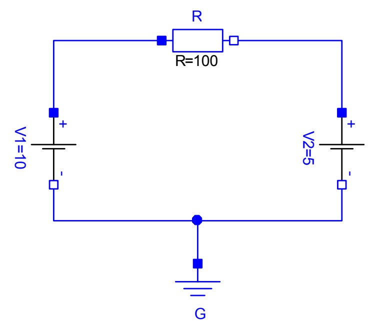
The current flows in this direction



Sink or Source?

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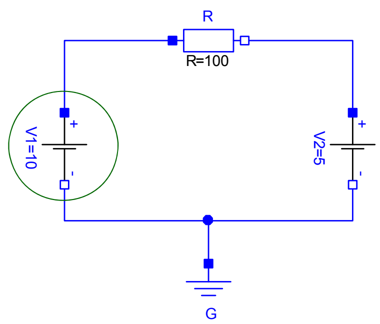
The current flows in this direction

Here, the current flows against the gradient of the potential voltage

➔ Source



Sink or Source?

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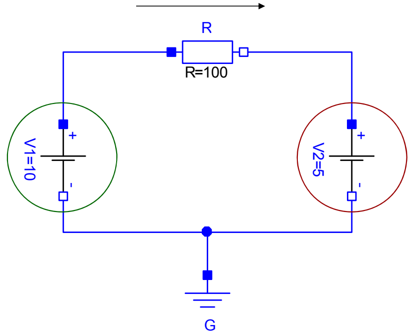
The current flows in this direction

Here, the current flows against the gradient of the potential voltage

➔ Energy Source



Here, the current flows along the gradient of the potential voltage

➔ Energy Sink

➔ A source of voltage is not necessarily a source of energy!

Bond Graphs

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We have seen that mechanical and electrical systems can be modeled the same way

- What about other physical domains?
- Can Kirchhoff`s Laws be generalized for the complete field of thermodynamics?

Bond Graphs

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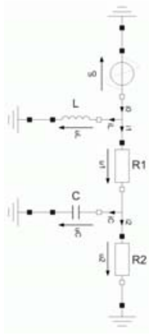
The answer is **bond graphs**.

- Here the complete system is abstracted by energy-flows.



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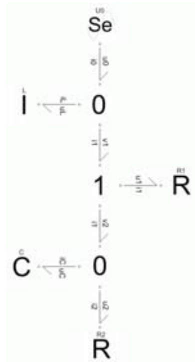

Bond Graphs

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For each physical domain, there is a specific pair of effort / flow variables

Domain	Potential	Flow
Translational Mechanics	Velocity: *v* [m/s]	Force: *f* [N]
Rotational Mechanics	Angular Velocity: ω [1/s]	Torque: τ [Nm]
Electrics	Voltage Potential v [V]	Current i [A]
Magnetics	Magnetomotive Force: Θ [A]	Time-derivative of Magnetic Flux: $\dot{\Phi}$ [V]
Hydraulics	Pressure p [Pa]	Volume flow rate $\dot{V}$ [m^3/s]
Thermal	Temperature T[K]	Entropy Flow Rate $\dot{S}$ [J/Ks]
Chemical	Chemical Potential: μ [J/mol]	Molar Flow Rate *v* [mol/s]

Bond Graphs

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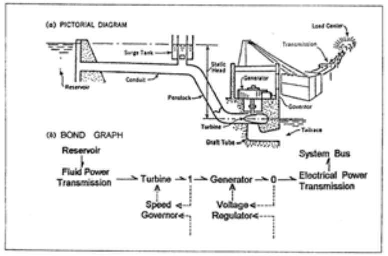
Bond graphs have been invented by Henry M. Paynter on April 24, 1959



Hydroelectric plant.

- Again, an actually trivial generalization of Kirchhoff's laws took more than a century to be developed.

Bond Graphs: Summary

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In this lecture, bond graphs are not the matter of subject, but we can profit from the major principle that underpins this methodology.

- For all physical domains, there is a correspondent pair of connector variables. Their product represents a flow of energy.
- The components all exhibit a certain energetic behavior.
- In this way, we do not have to acquire the physical knowledge domain by domain. Instead we apply the general principles of thermodynamics.

Conservation of Energy

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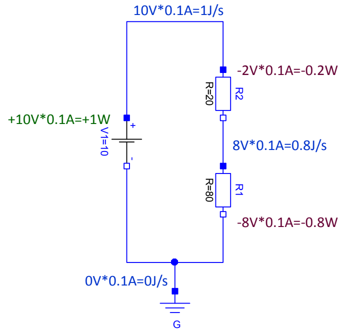
By modeling with energy flows, we can profit form the general laws of thermodynamics.



- The first law of thermo-dynamics states that within a closed system, the total amount of energy remains constant.
- This means that the sum of all powers quantities across the components must be zero.

Energetic correct behavior

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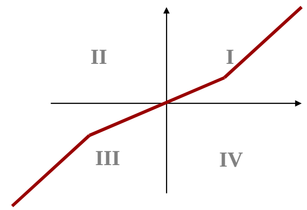
Any dissipative component represents a relates the flow *F* with a difference of potentials Δ*P*.

Δ*P* = f(*F*)



- The corresponding function f(…) must be located in first and third quadrant (and cross the origin).

Energetic correct behavior

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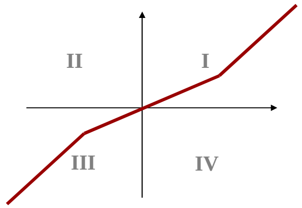
Any storage component relates one of the two variables with the time-derivative of its partner.

dΔ*P*/dt = f(F)

or

d*F*/dt = f(Δ*P*)



- Also here: the corresponding function f(…) must be located in first and third quadrant (and cross the origin).

Multi-Domain Modeling

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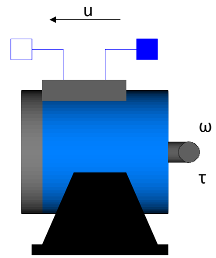
Using energy flows, we can also model across multiple domains

- An electrical engine represents a transformer from electrical energy to mechanic (rotational) energy. Energy is conserved.

τ = K·i

K · ω = u



- K is the Motor-Torque Constant

Multi-Domain Modeling

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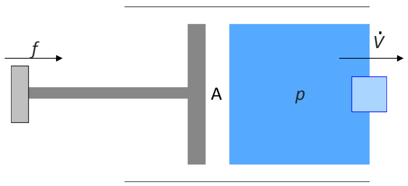
Using energy flows, we can also model across multiple domains

- A piston represents a transformer (more precisely: a gyrator) from the mechanical domain into the hydraulic domain. Also here, energy is conserved.

$$p \cdot A = f$$
$$\dot{V} = v \cdot A$$



- A is the area of the piston

The 2nd Law of Thermodynamics

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- Ideally, any form of energy can be completely transformed into any other. (Practically, all transformations involve dissipation.)
- The dissipation of energy represents the transformation of energy into thermal energy.
- But there is one important exception: The 2nd law of thermodynamics states that entropy can only increase.
- The thermal domain possesses the flow of entropy as connector variable. This means, that for any thermal sub-system the inflow must be equal or greater than the outflow.

The 3rd Law of Thermodynamics

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- Thermal energy can only be transformed into other forms of energy up to a limited extent.
- In order to transform thermal energy into any other form, we need a temperature gradient between two reservoirs T_{cold} and T_{hot}.
- The precise limit of the efficiency is determined by the Carnot Factor. This is the 3rd law of Thermodynamics.

$$\eta_C = 1 - T_{cold}/T_{hot}$$

(Temperature in Kelvin)

- Since $T_{cold} > 0 \rightarrow \eta_C < 1$

Summary

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- All physical connections can be represented by a pair of a potential variable and a flow variable whose product represents energy flow.
- Using this knowledge, the equations for the connections can be automatically generated.
- All components exhibit a certain energetic behavior. Once we understand the energetic behavior, we can apply it in various physical domains.
- Interaction between domains is represented by a transformation of energy.

Summary

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- Next week, we are going to learn how to punch all this into a computer!
- Don't worry if you haven't understood every single component equation. We will look at the modeling of electrical and mechanical systems in depth.

Questions ?