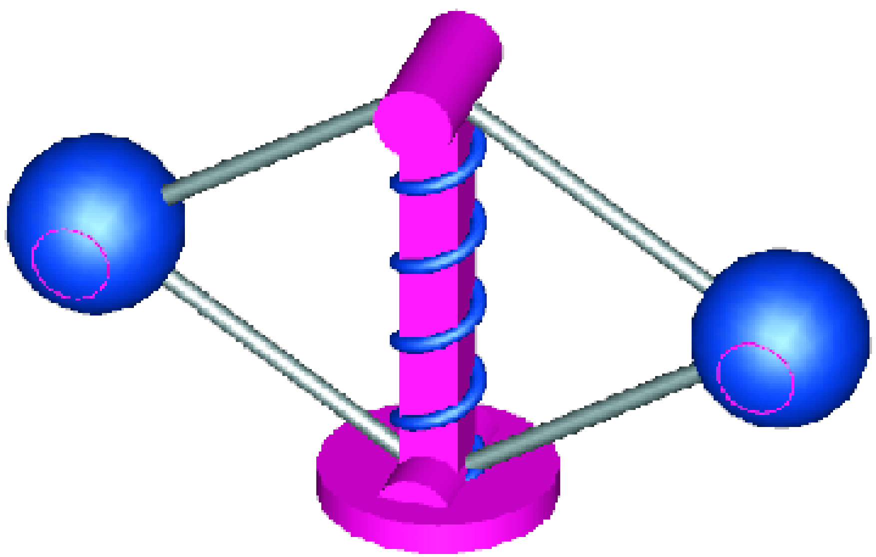
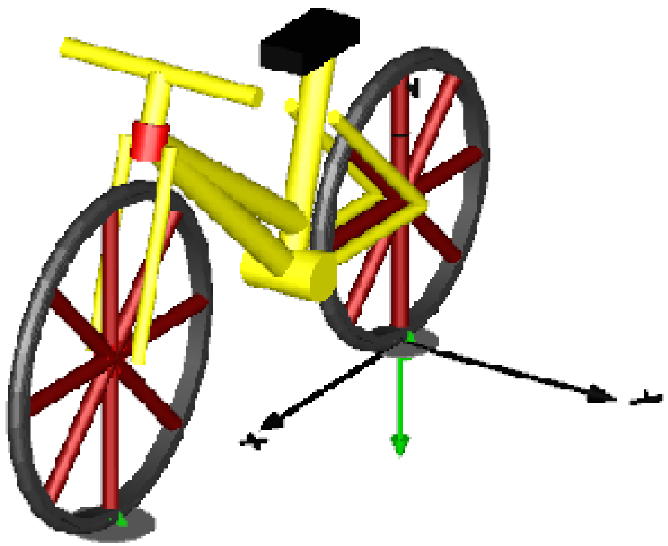
Virtual Physics
Equation-Based Modeling

TUM, November 25, 2014

3D Mechanics, Part II





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3D Mechanics

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In this lecture, we look at the modeling of 3D mechanical systems.

- 3D mechanical models look superficially just like planar mechanical models. There are additional types of joints, but other than that, there seem to be few surprises.

- Yet, the seemingly similar appearance is deceiving. There are a substantial number of complications that the modeler has to cope with when dealing with 3D mechanics. These are the subject of this lecture.

3D Mechanics

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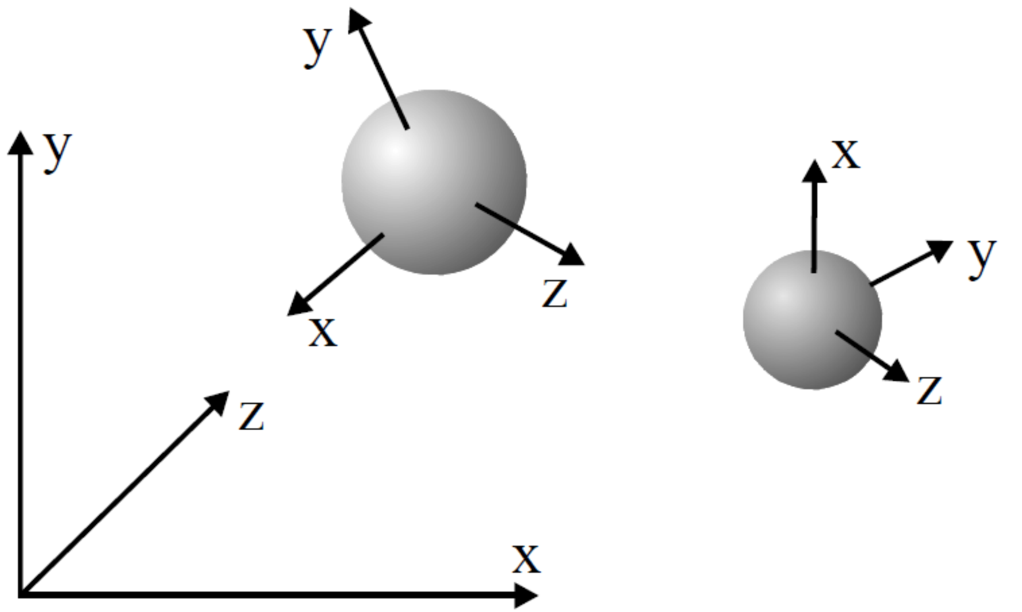
Essentially, there are 3 major difficulties we have to cope with:

1. There are multiple ways to express the orientation of a body in three dimensional space.

2. In planar mechanics, all potential variables could be expressed in one common coordinate system: The inertial system. In 3D-mechanics, such an approach is unfeasible.

3. The set of connector variables contains a redundant set of variables. This causes severe problems for the formulation of kinematic loops.

Fundamental set of equations



- In planar mechanics, all connector variables are resolved w.r.t. the inertial coordinate system.
- In 3D-mechanics, we will refer also to the body system. A coordinate system that is attached to each body.
- Notation The index *0* indicates that a vector is resolved w.r.t. to the ineratial system. The index *body* indicates that is resolved w.r.t. to its body system.

Rotation Matrix

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- The rotational matrix can be used to transform between these coordinate systems. For instance

$$\mathbf{R}\boldsymbol{\omega}_0 = \boldsymbol{\omega}_{body}$$

$$\boldsymbol{\omega}_0 = \mathbf{R}^T\boldsymbol{\omega}_{body}$$

- Repetition: The rotational matrix is the one to integrate:

$$\tilde{\omega}_0\mathbf{R} = \mathbf{R}\tilde{\omega}_{body} = \dot{\mathbf{R}}$$

Fundamental set of equations

- The fundamental set of equations can be formulated in the inertial system:

$$\mathbf{f}_0 = m \cdot \mathbf{a}_0$$

$$\mathbf{t}_0 = \mathbf{J}_0 \dot{\boldsymbol{\omega}}_0$$

- In planar mechanics, the rotational inertia was represented by a simple scalar I. In 3D mechanics, it is represented by a 3D matrix **J**: the inertia tensor.
- However, $\mathbf{J_0}$ is not a constant during motion since it depends on the orientation of the body.

Fundamental set of equations

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- In the body-system, the inertia tensor $\mathbf{J}_{body}$ is constant. Hence we can transform the law into the body system:

$$\mathbf{t}_0 = \frac{d}{dt}\left(\mathbf{R}^T \mathbf{J}_{body} \boldsymbol{\omega}_{body}\right)$$

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$$\mathbf{t}_0 = \dot{\mathbf{R}}^T \mathbf{J}_{body} \boldsymbol{\omega}_{body} + \mathbf{R}^T \mathbf{J}_{body} \dot{\boldsymbol{\omega}}_{body}$$

Fundamental set of equations

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$$\mathbf{R}^T \mathbf{t}_{body} = \mathbf{R}^T \tilde{\omega}_{body} \mathbf{J}_{body} \boldsymbol{\omega}_{body} + \mathbf{R}^T \mathbf{J}_{body} \mathbf{z}_{body}$$

Fundamental set of equations

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$$\mathbf{t}_0 = \frac{d}{dt}\left(\mathbf{R}^T \mathbf{J}_{body} \boldsymbol{\omega}_{body}\right)$$

$$\mathbf{t}_0 = \dot{\mathbf{R}}^T \mathbf{J}_{body} \boldsymbol{\omega}_{body} + \mathbf{R}^T \mathbf{J}_{body} \dot{\boldsymbol{\omega}}_{body}$$

$$\mathbf{R}^T \mathbf{t}_{body} = \mathbf{R}^T \tilde{\omega}_{body} \mathbf{J}_{body} \boldsymbol{\omega}_{body} + \mathbf{R}^T \mathbf{J}_{body} \mathbf{z}_{body}$$

$$\mathbf{t}_{body} = \boldsymbol{\omega}_{body} \times \mathbf{J}_{body} \boldsymbol{\omega}_{body} + \mathbf{J}_{body} \mathbf{z}_{body}$$

Fundamental set of equations

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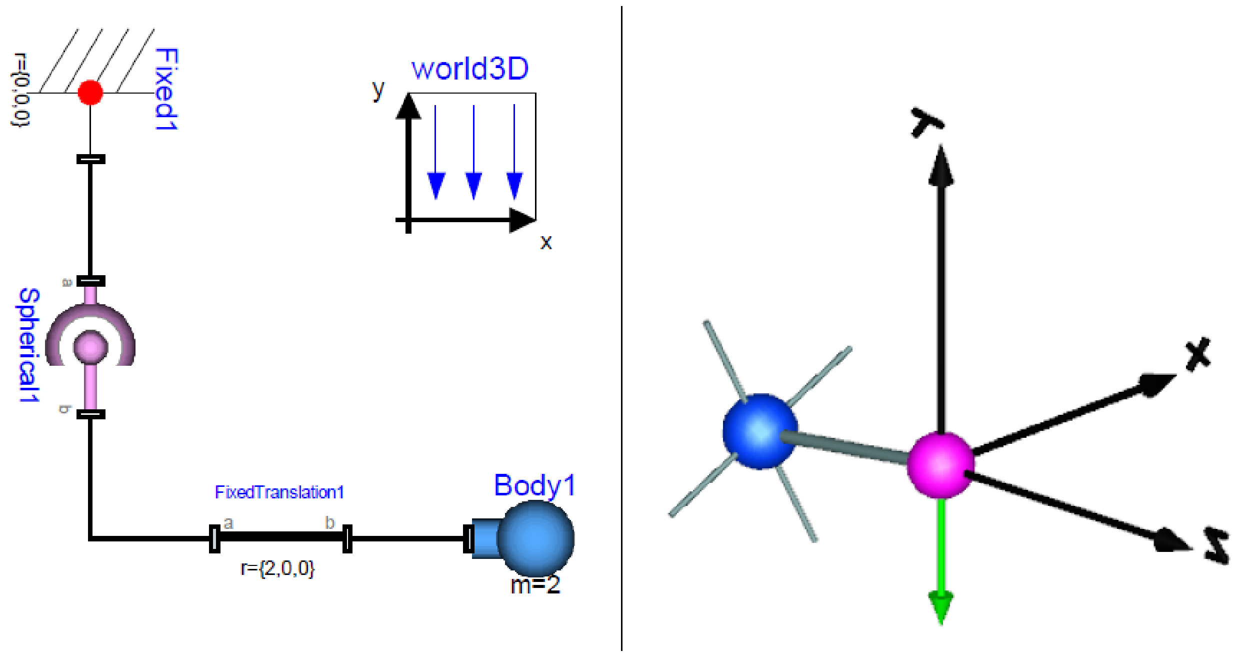
- In the body-system, the inertia tensor $\mathbf{J}_{body}$ is constant. Hence we can transform the law into the body system:
- An additional term for the torque occurs: The gyroscopic torque.
- This torque is a pseudo-torque that resulted out of the transformation into the body system.

$$\mathbf{t}_{body} = \boxed{\boldsymbol{\omega}_{body} \times \mathbf{J}_{body}\boldsymbol{\omega}_{body}} + \mathbf{J}_{body}\mathbf{z}_{body}$$

Selection of Method

- We have observed the (highly non-intuitive) behavior of the gyroscopic effect, already last lecture:



Connector Design

- The translational components can be more conveniently described in the inertial system.

- The rotational components are preferably resolved w.r.t. to the body system.

Connector Design

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- In the MultiBody library, the connector is designed as follows:
- Vectors and matrices are supported natively by Modelica and used for the connector variables.

```
connector Frame

  SI.Position r_0[3];
  Real T[3, 3];
  SI.AngularVelocity w[3]

  flow SI.Force f[3];
  flow SI.Torque t[3];

end Frame;
```

Connector Design

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- In the MultiBody library, the connector is designed as follows:

- Resolved w.r.t. to the inertial system:

  r_0, T

- Resolved w.r.t. to the body system (T):

  w, t, and f (why ever...)

```
connector Frame

  SI.Position r_0[3];

  Real T[3, 3];

  SI.AngularVelocity w[3]

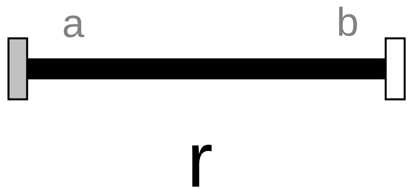

  flow SI.Force f[3];

  flow SI.Torque t[3];

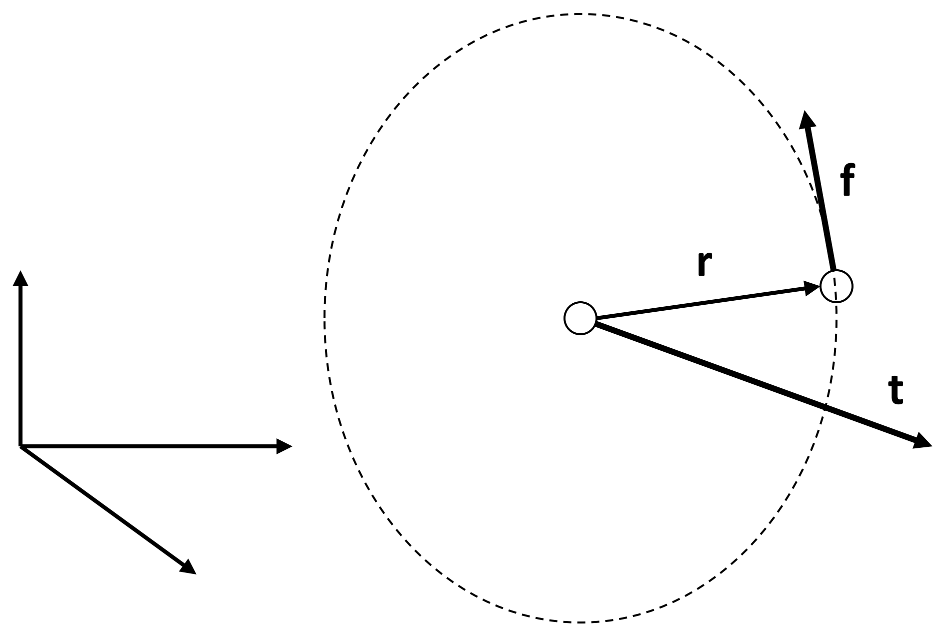
end Frame;
```

An example component

Let us look at the fixed translation component:



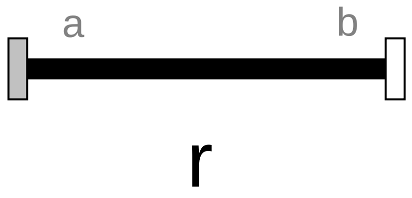
- It essentially represents the lever principle.



$$t = r \times f$$

An example component

Let us look at the fixed translation component:



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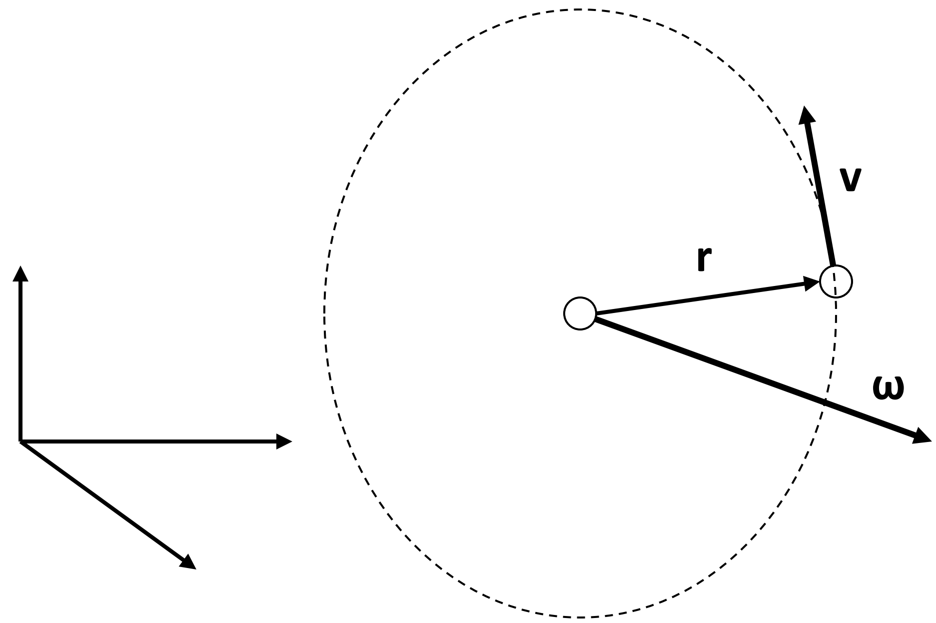
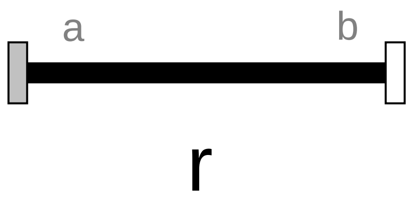


$$\mathbf{v} = \mathbf{r} \times \boldsymbol{\omega}$$

An example component

Let us look at the fixed translation component:



- It essentially represents the lever principle.

```
model FixedTranslation
  parameter SI.Position r[3] = {0,0,0};

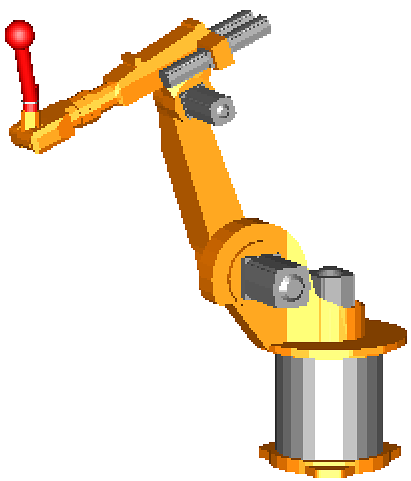
  frame_b.r_0 = frame_a.r_0 + transpose(frame_a.T)*r;
  frame_b.T = frame_a.T;
  frame_b.w = frame_a.w;

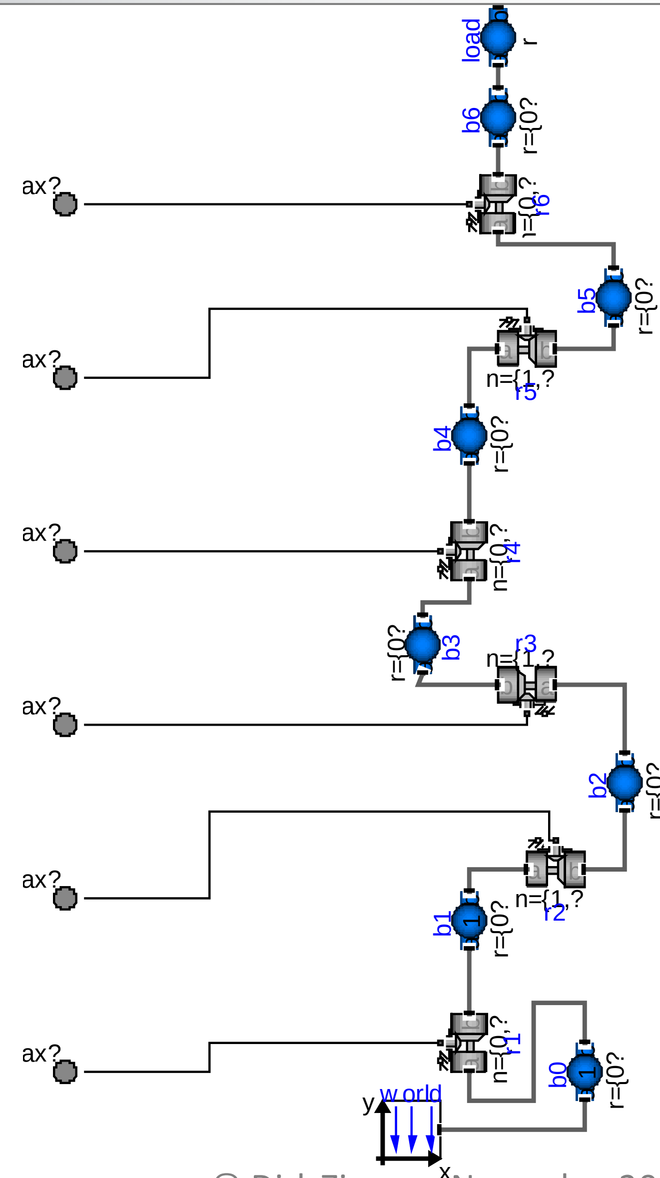
  zeros(3) = frame_a.f + frame_b.f;
  zeros(3) = frame_a.t + frame_b.t + cross(r, frame_b.f);

end FixedTranslation;
```

An example system: The robot

- Here, the multibody components are used to assemble a robot.





An Example System: The Robot

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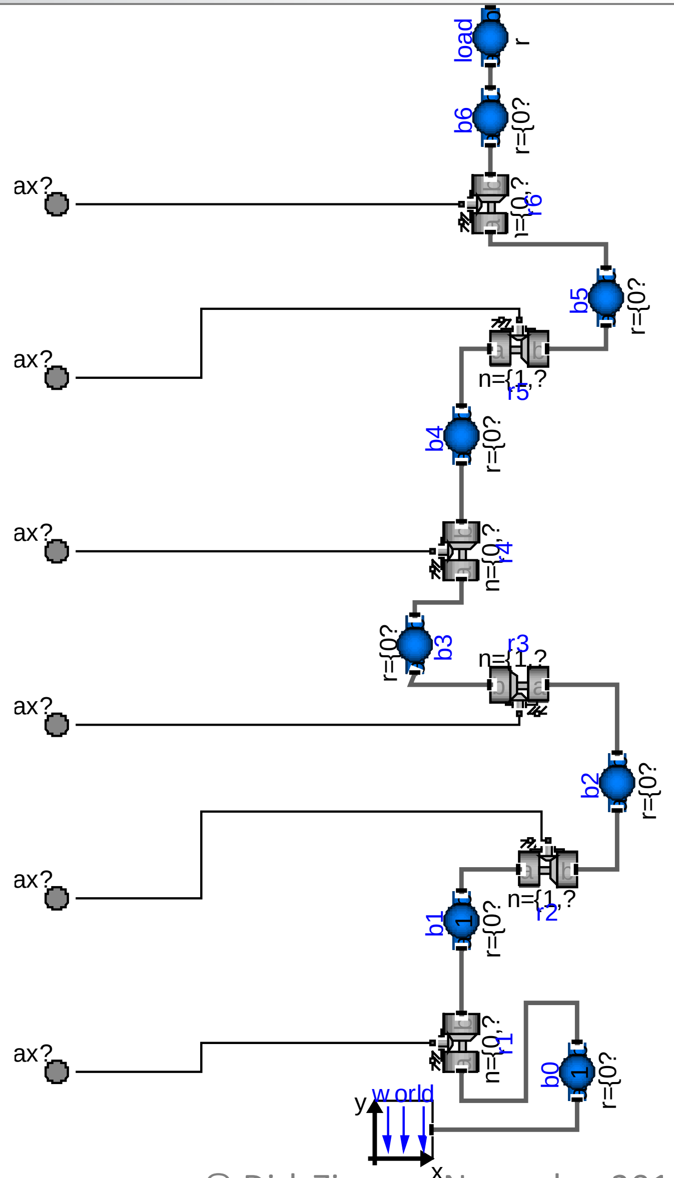
- Here, the multibody components are used to assemble a robot.

- It essentially consists out of fixed translations combined with body parts and actuated revolute joints.



Redundant Connector Variables

- The potential variables of the Multibody connector are highly redundant.
- Only 3 variables are sufficient to describe the 3D-rotation.
- But the connector contains 3*3 + 3 = 12 potential variables for the rotational part.

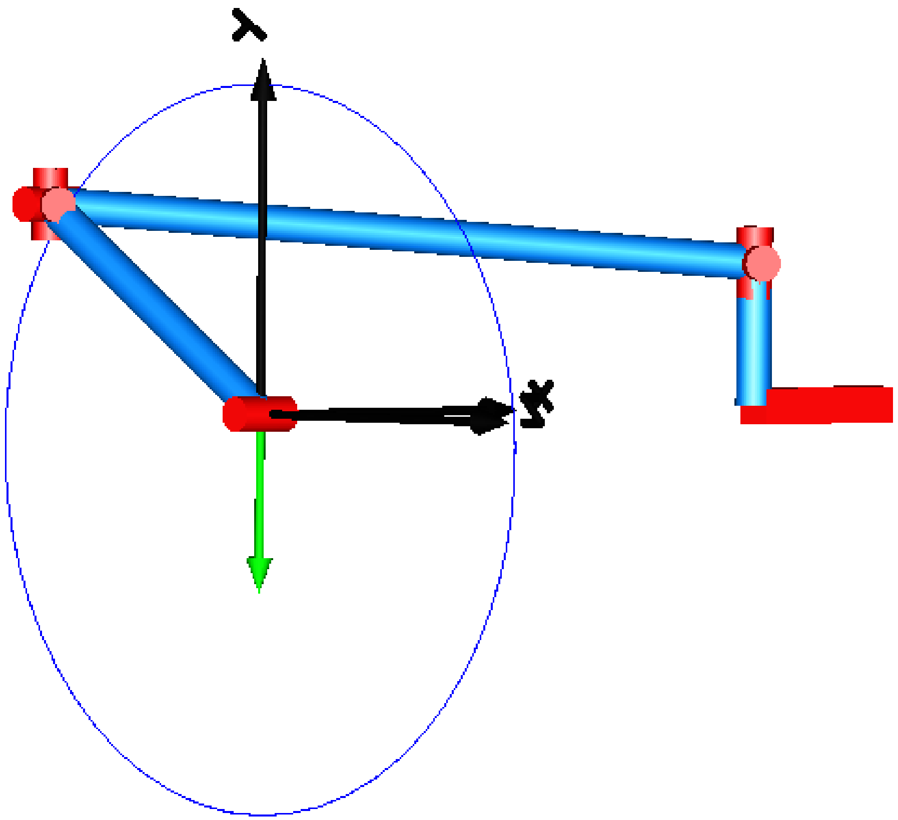
```
connector Frame

  SI.Position r_0[3];
  Real T[3, 3];
  SI.AngularVelocity w[3]

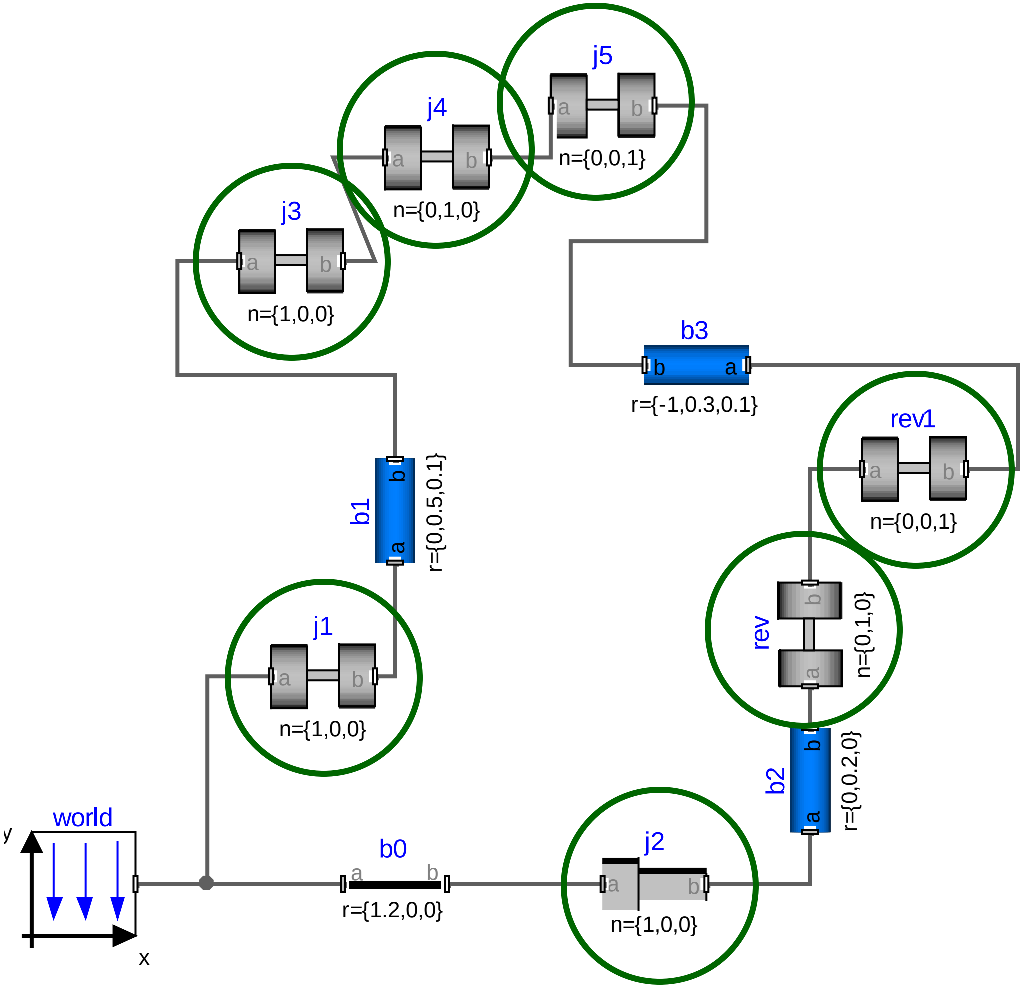
  flow SI.Force f[3];
  flow SI.Torque t[3];

end Frame;
```

Kinematic Loops



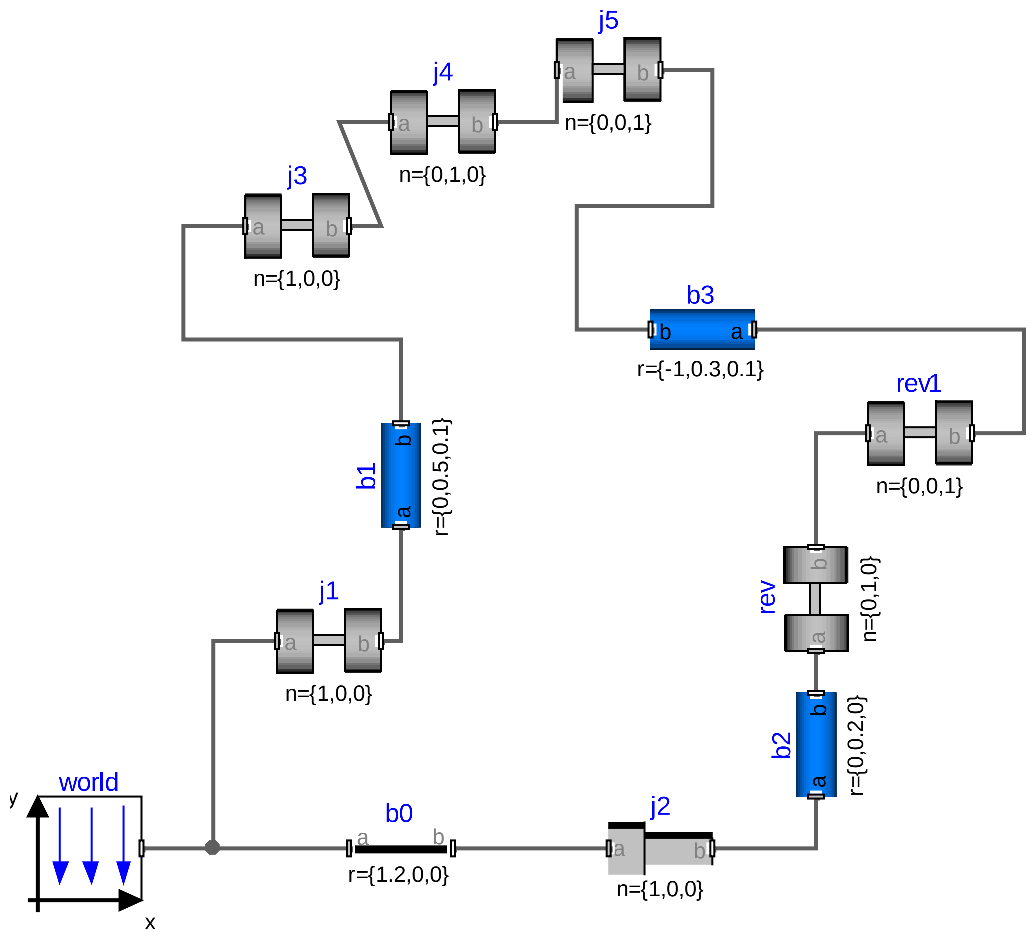
Kinematic Loops



7 degrees of freedom – 6 constraint equations = 1 degree of freedom

Kinematic Loops

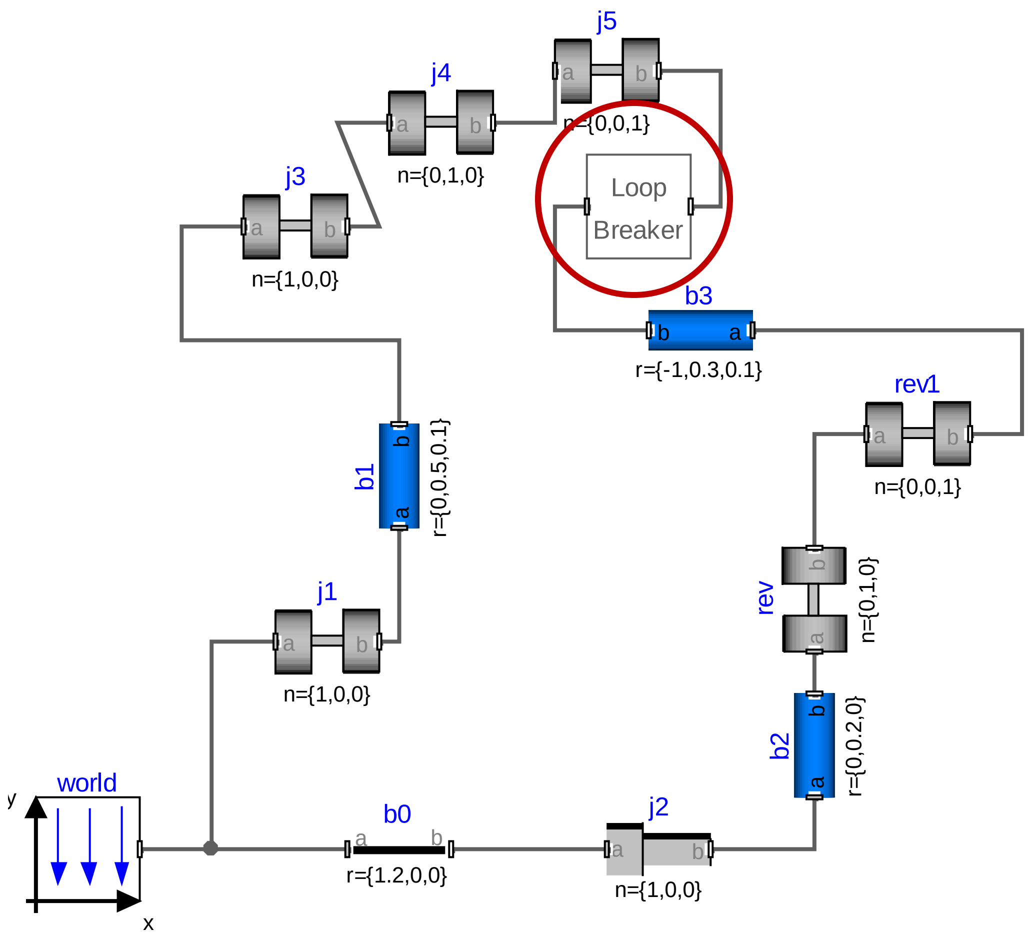
- This redundancy causes severe problems in case of kinematic loops.
- Closing a kinematic loop establishes 6 constraint equations.
- But the redundant connector set leads to 15 constraint equations (these are 9 too many).



Kinematic Loops

- In the "old days", the loops had to manually closed with the aid of a loop-breaker.

- The loop-breaker is a model that contains just the necessary 6 constraint equations
  (and the balance of force and torque, naturally)



Kinematic Loops

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```
model LoopBreaker
  Interfaces.Frame_a frame_a;
  Interfaces.Frame_b frame_b;

equation
  frame_a.r_0 frame_b.r_0;
  cross(frame_a.T[1, :], frame_a.T[2, :])*frame_b.T[2, :] = 0;
  -cross(frame_a.T[1, :],frame_a.T[2, :])*frame_b.T[1, :] = 0;
 frame_a.T[2, :]*frame_b.T[1, :] = 0
 frame_a.f + frame_b.f = zeros(3);
 frame_a.t + frame_b.t = zeros(3);
end LoopBreaker
```

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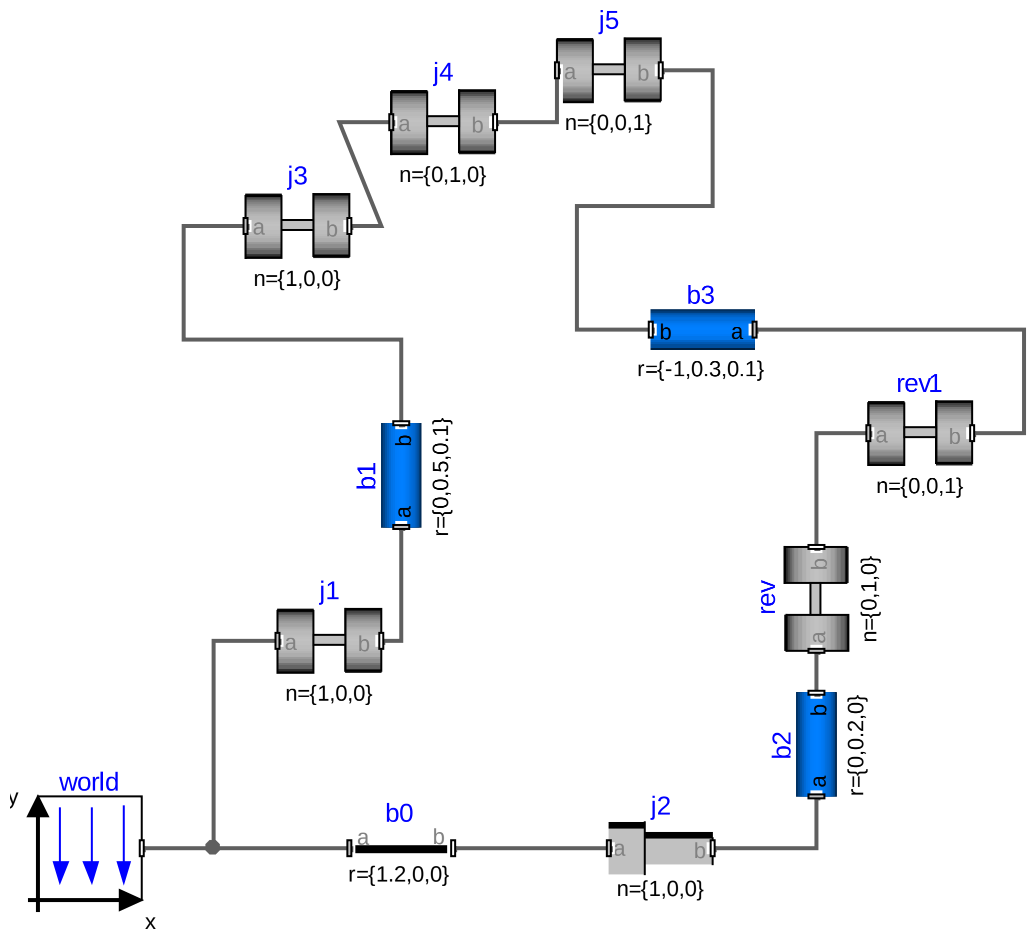
 frame_a.T[2, :]*frame_b.T[1, :] = 0
 frame_a.f + frame_b.f = zeros(3);
 frame_a.t + frame_b.t = zeros(3);
end LoopBreaker
```

Kinematic Loops

- Nowadays, the loop breaker is not necessary anymore.
- The process has been automated (by introducing a whole new set of irritating language constructs).



Planar kinematic loops

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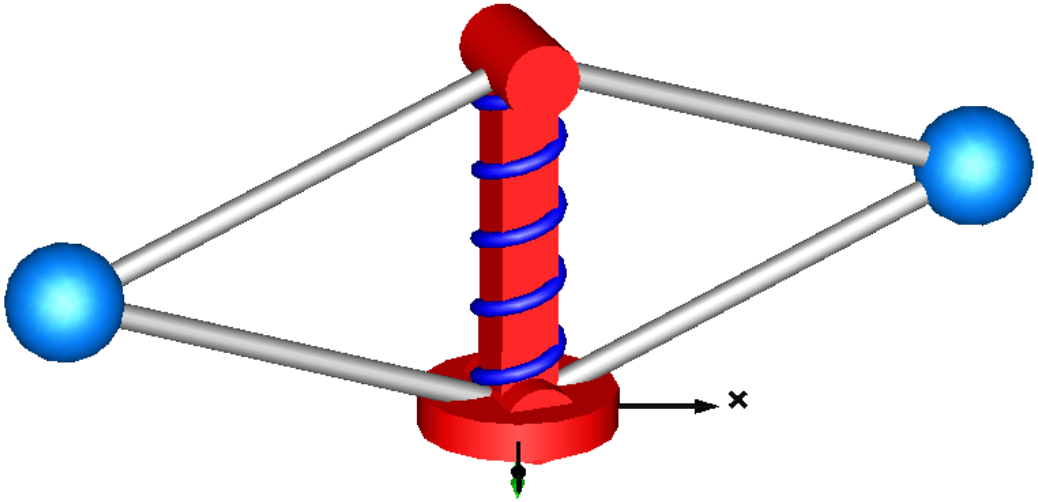
- Another special case are planar kinematic loops within 3D mechanics.
- Even if we apply the correct set of constraint equations, we get a singular system.
- Let us look at an example…

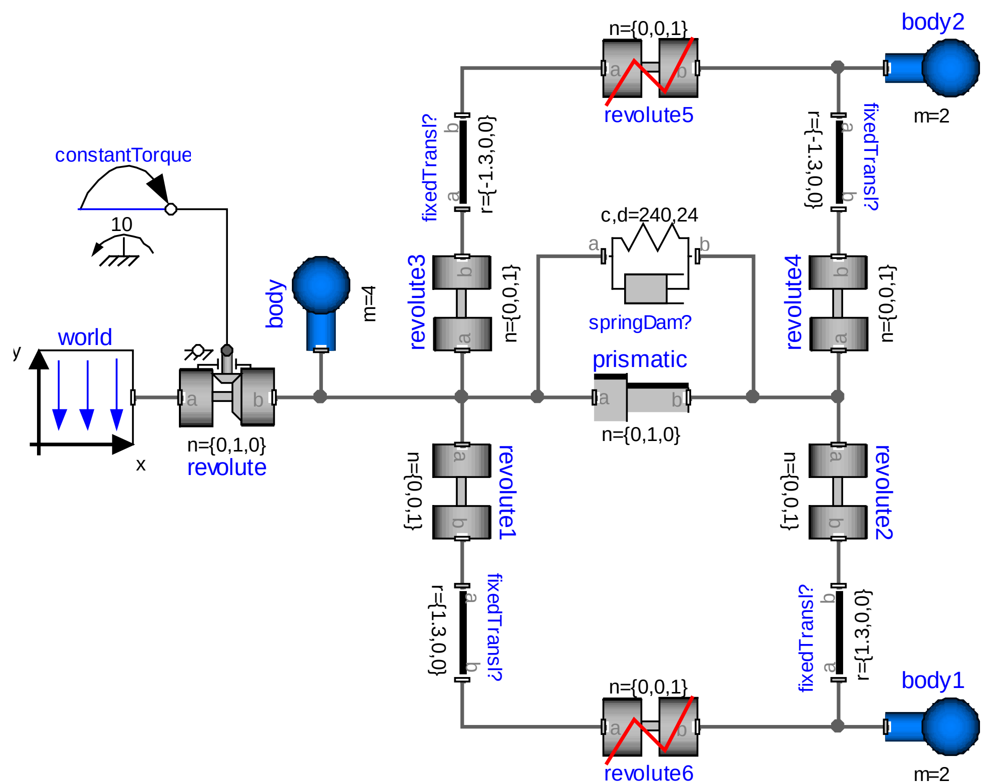


Planar kinematic loops

- The problem is the following:

- There are two planar closed kinematic loops each defined by three revolute joints and a prismatic joint.

- Two revolute joints with the same rotation axis suffice to restrict the freedom of motion to a single axis. The constraint of the third revolute joint is therefore superfluous, which leads to an additional redundancy



Planar kinematic loops

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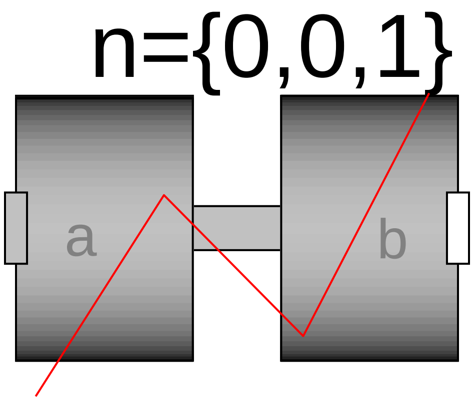
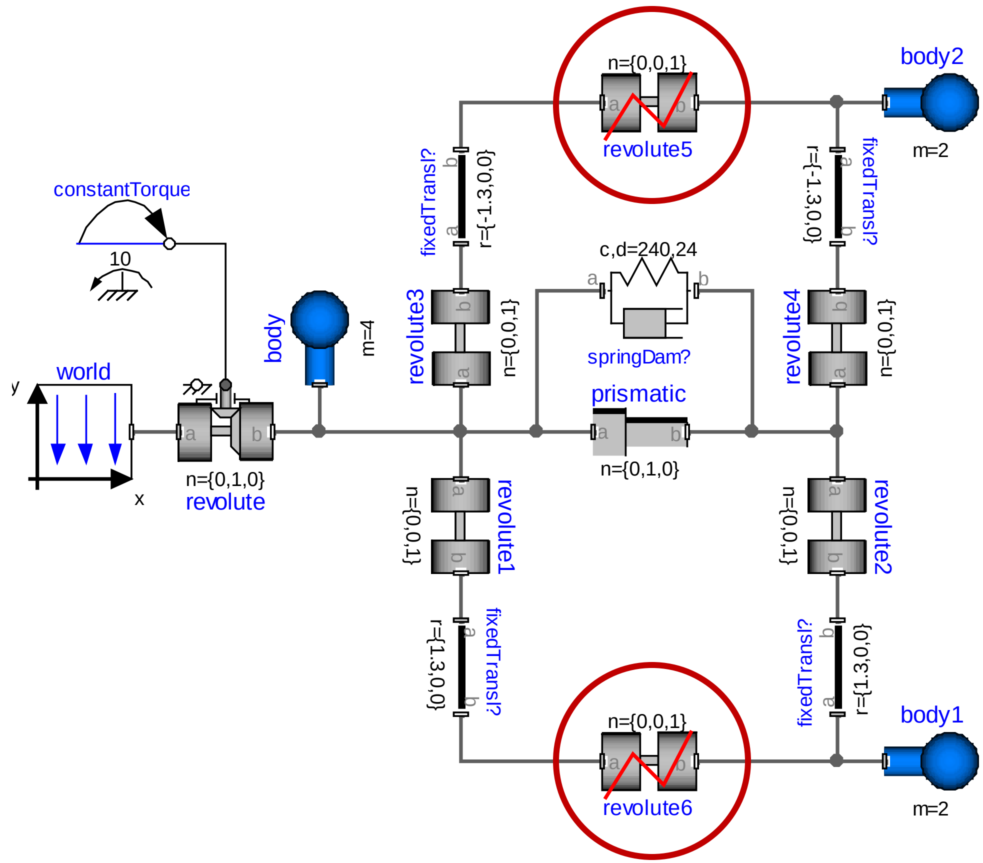
- To this end, there is a special revolute joint to cut the planar loop.





Questions ?