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Virtual Physics

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Exercise 10: Stability Analysis (Solution)

Task 1:

Solution starts with linearization around $\boldsymbol{x}_P$ with $\boldsymbol{x} = \boldsymbol{x}_p + \Delta\boldsymbol{x}$ and $\boldsymbol{x}$ representing the vector $\binom{x}{y}$

$$\binom{\dot{x}}{\dot{y}} = 0 + \begin{pmatrix} 0 & \frac{1}{4} \\ -2x_p - 1 & -1 \end{pmatrix} \binom{\Delta x}{\Delta y}$$

if x_P represents an equilibrium point

Characteristic polynomial to retrieve eigenvalues:

$$\begin{pmatrix} 0 - \lambda & \frac{1}{4} \\ -2x_p - 1 & -1 - \lambda \end{pmatrix}$$

$$(0 - \lambda)(-1 - \lambda) + \frac{1}{2}x_p + \frac{1}{4} = 0$$

At $x = 0$:

$$\lambda^2 + \lambda + \frac{1}{4} = 0 = \left(\lambda + \frac{1}{2}\right)\left(\lambda + \frac{1}{2}\right)$$

…results in two negative eigenvalues and hence the system is approx. stable around this equilibrium point.

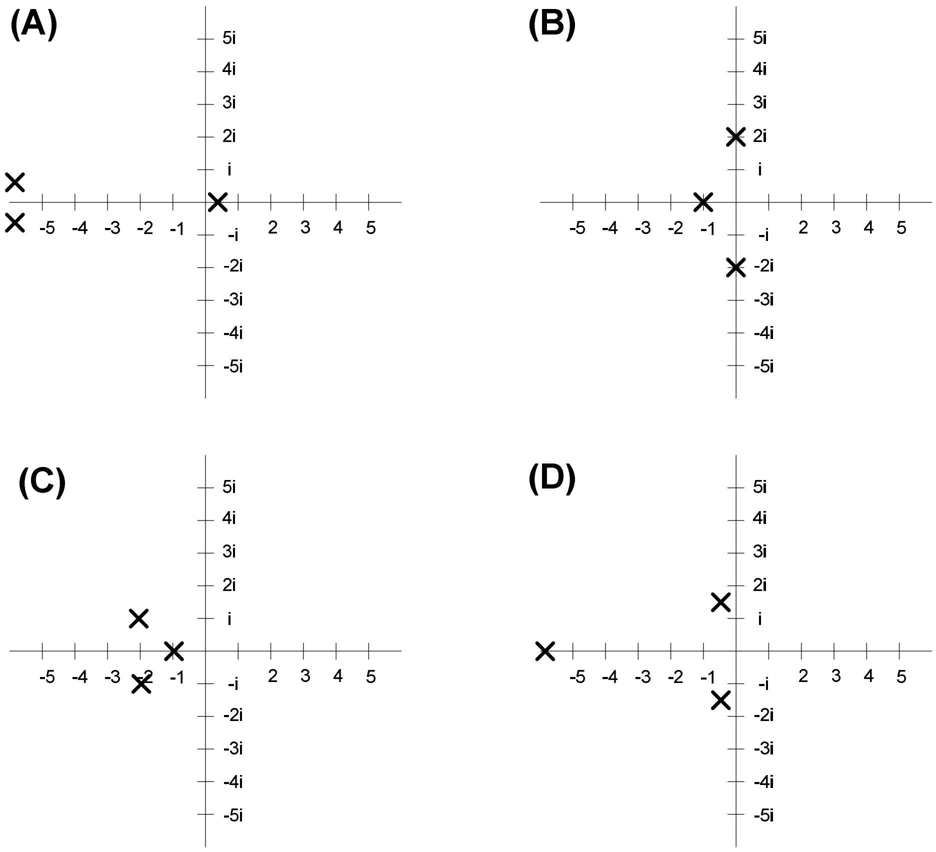
At $x = -1$:

$$\lambda^2 + \lambda - \frac{1}{4} = 0$$

The polynomial is an upwards shaped parabola with a negative value for $\lambda = 0$ and hence will have a zero-crossing (aka solution) for positive λ. There is a positive eigenvalue. Hence the system is unstable around this equilibrium point.

Task 2:

The eigenvalues of four linear systems (dx/dt = Ax) are depicted.



Mark what is true (12 points):

	A	B	C	D
The system is stable			X	X
The system is unstable	X			
The system is marginally stable		X		
The system is stiff				X
The system is numerically stable for Forward Euler with step-size h = 0.5			X	