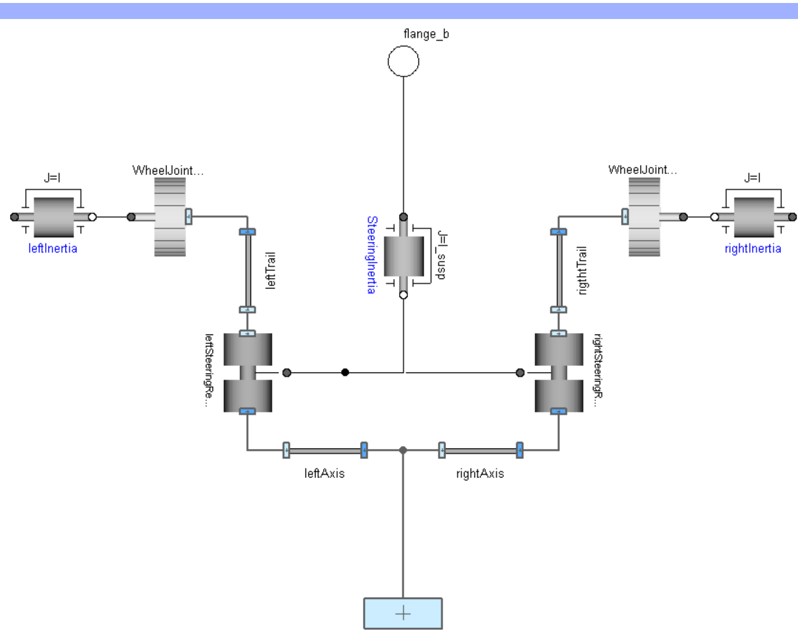
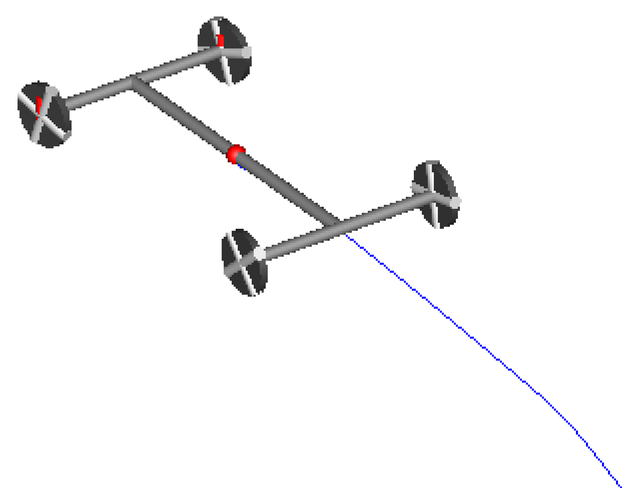
Virtual Physics
Equation-Based Modeling

TUM, December 20, 2022

Two-Track Car Model





Dr. Dirk Zimmer

German Aerospace Center (DLR), Robotics and Mechatronics Centre

Two-Track Model

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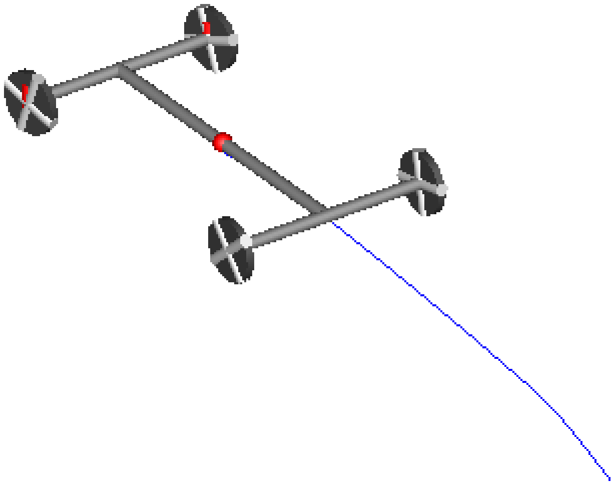
In this lecture, let us look at the modeling of a two-track car model:

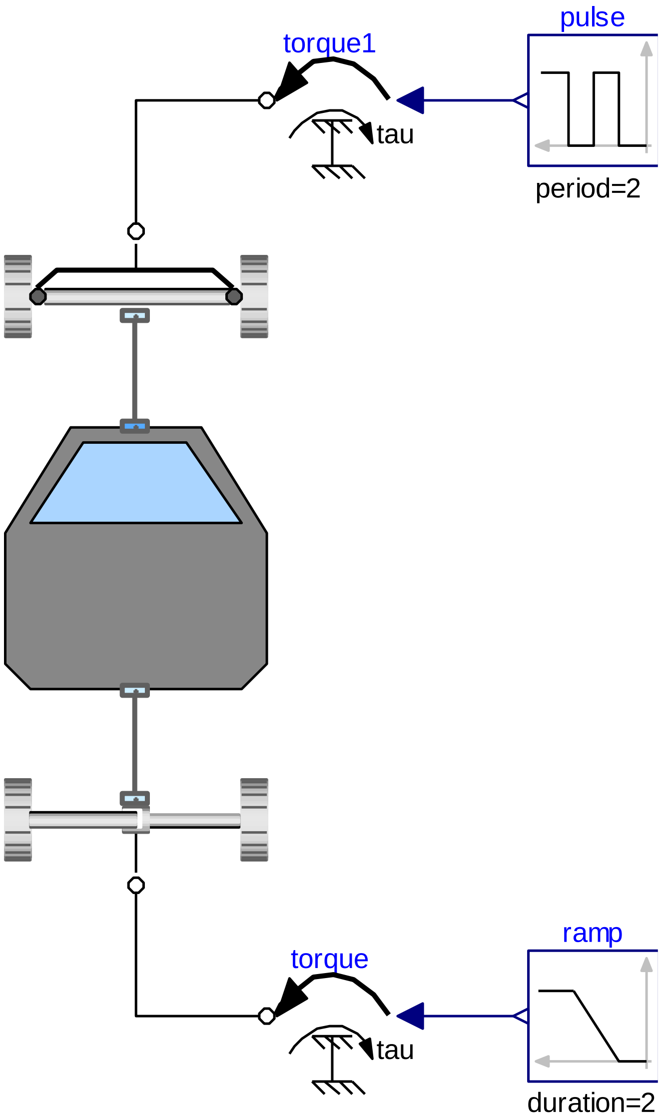
- This is still a planar mechanical model
- It will be later enhanced by a few 3D-elements.



Two Track Model

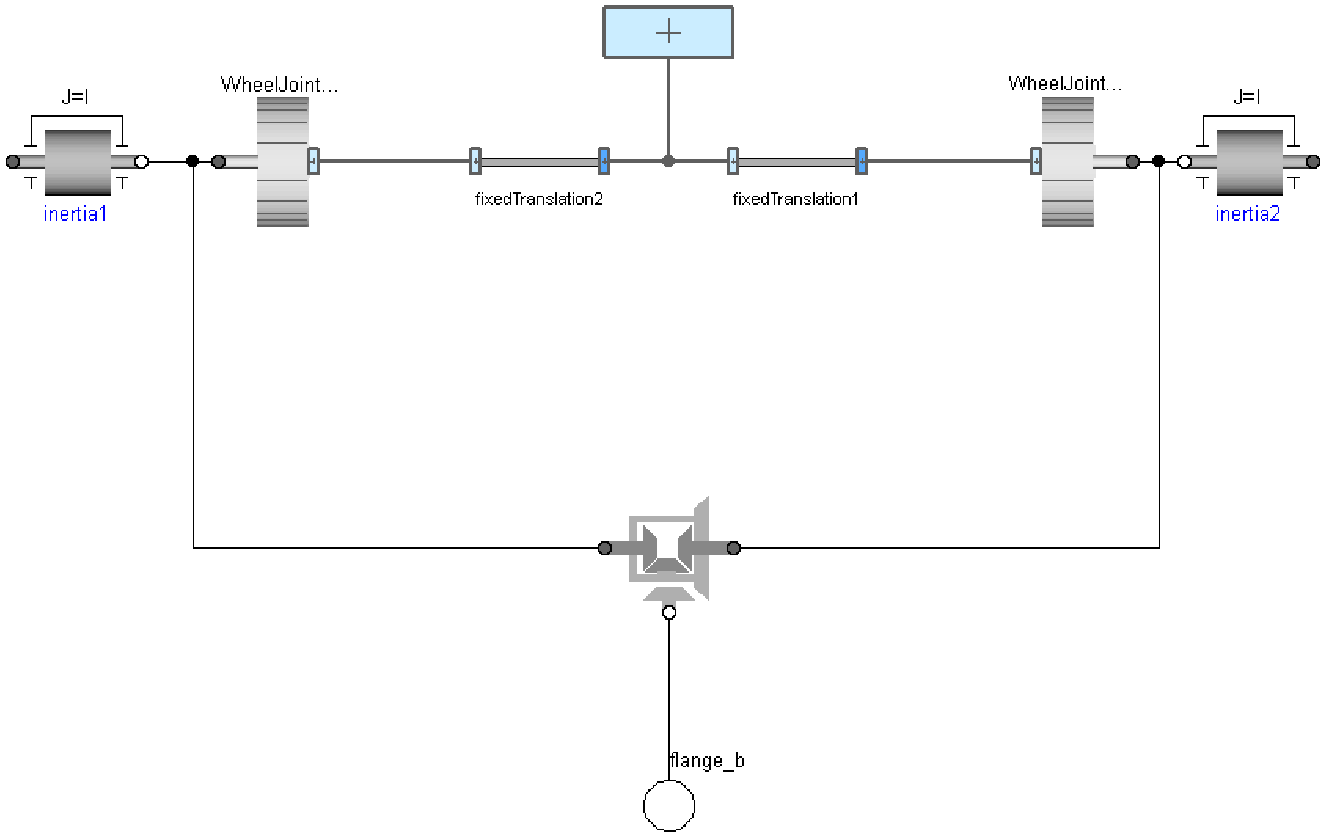
In this lecture, let us look at the modeling of a two-track car model:

- It essentially consists in three parts:
- The rear axis
- The front axis
- The chassis



Rear Axis

- The rear axis connects the rotation of the two wheels by a differential.
- The wheels are dry-friction based.



Front Axis

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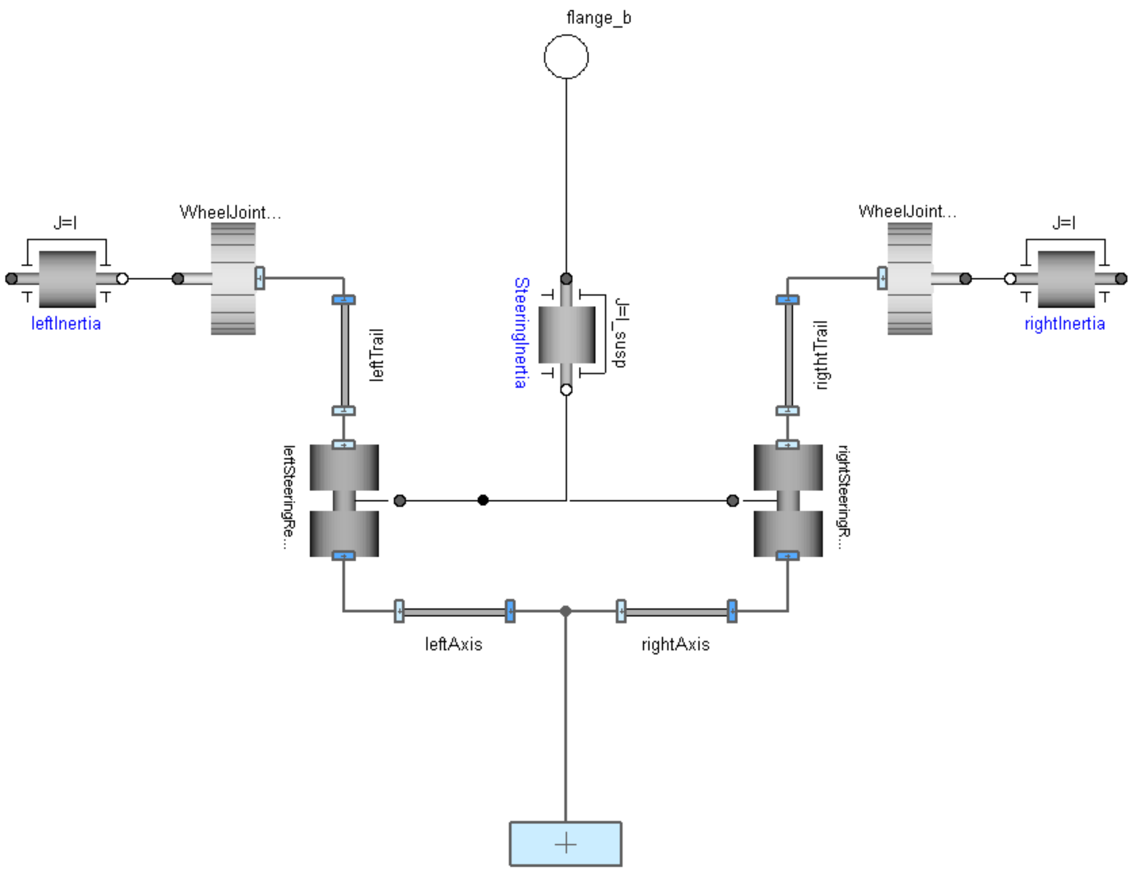
- In the front axis, the steering revolute are rigidly connected.
- The whole steering mechanism shares one common inertia.



Chassis

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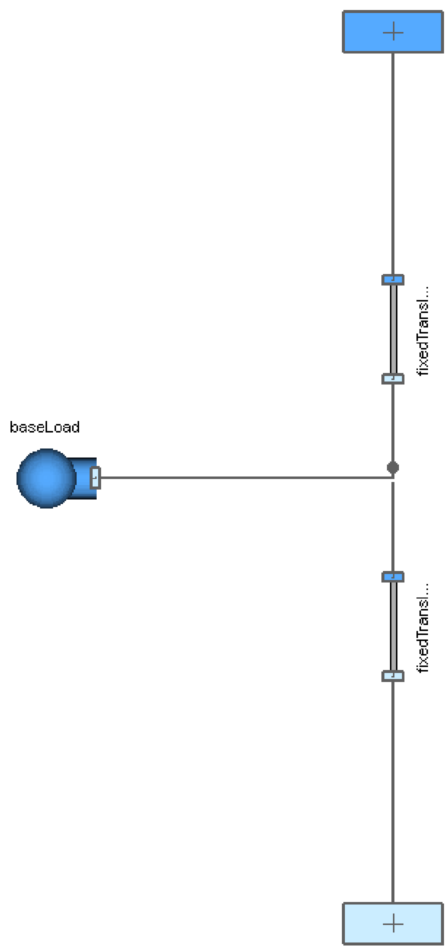
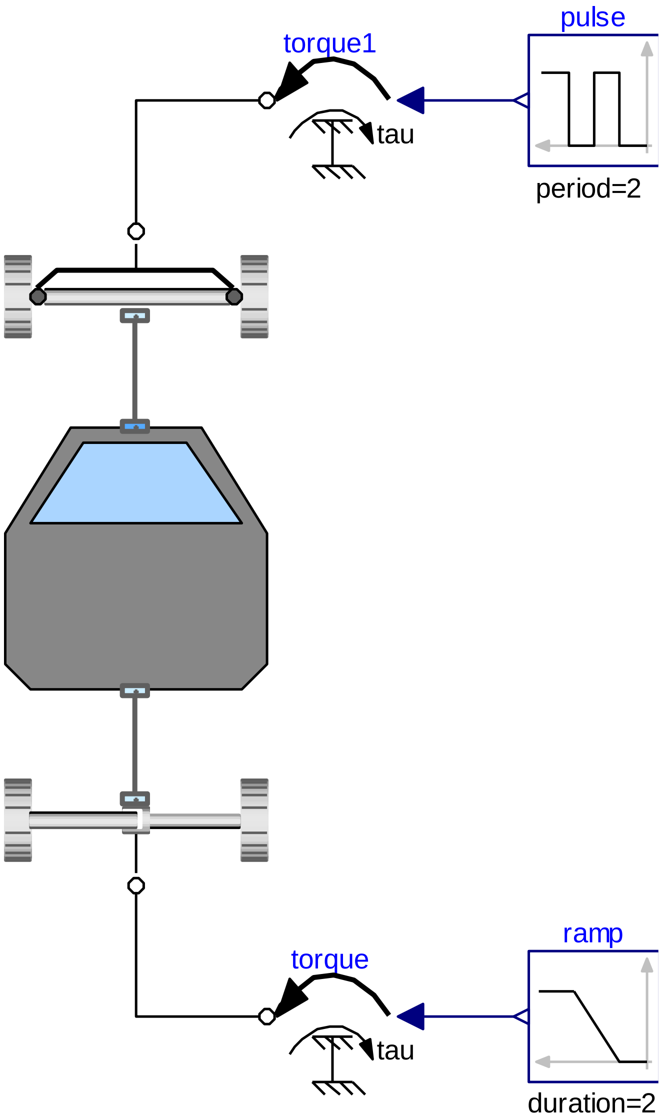
- Modeling the chassis represents a triviality.
- It simply models the “geometry” and puts a mass with inertia at the center.



Experiment

- The experiment on the right ramps up the driving torque
- A pulse wise torque acts on the steering and leads to sudden deflections.



Experiment Results

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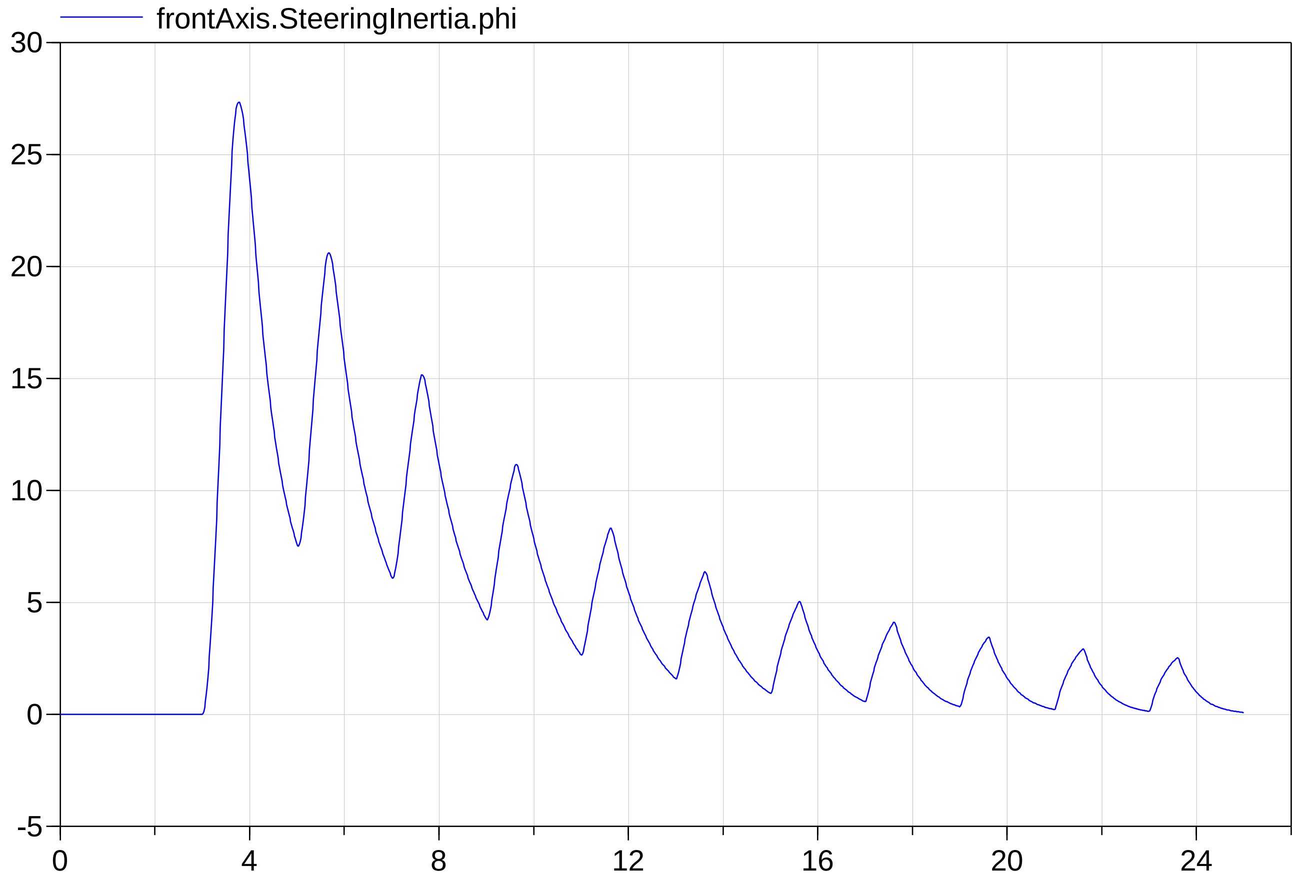
- The car is accelerating and the pulsed torque leads to smaller steering angles.



3D Chassis

On top of the classis two-track model, we want to build a 3D-chassis that can tilt in two directions (roll and pitch)

- We need a conversion interface from 2D to 3D mechanics.
- We need to model the balancing behavior.

From 2D to 3D

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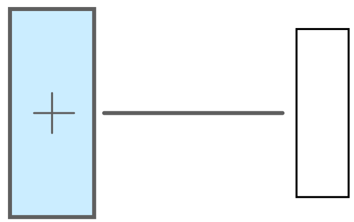
Let us look at the interface from 2D to 3D:



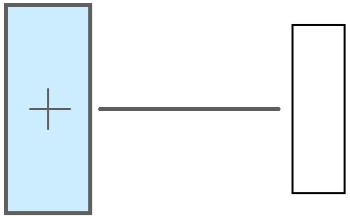
- From 3D perspective this element is like a joint with 3 degrees of freedom.

- Or alternatively, the “joint” establishes 3 holonmic constraints by resticting the motion to a single plane.

```
model PlanarToMultiBody

  Frame_a frame_a;
  MB.Interfaces.Frame_b frame_b;

protected
  SI.Force fz "Normal Force";
  SI.Force f0[3] "Force vector";

equation
  frame_a.x = frame_b.r_0[1];
  frame_a.y = frame_b.r_0[2];
  0 = frame_b.r_0[3];
  frame_b.R =
    MB.Frames.planarRotation({0,0,1},
    frame_a.phi, der(frame_a.phi));

  f0 = {frame_a.fx, frame_a.fy, fz};
  f0*frame_b.R.T + frame_b.f = zeros(3);
  frame_a.t + frame_b.t[3] = 0;

  Connections.root(frame_b.R);
end PlanarToMultiBody
```

From 2D to 3D

Let us look at the interface from 2D to 3D:



- We prescribe the position…
- …and the orientation

```
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From 2D to 3D

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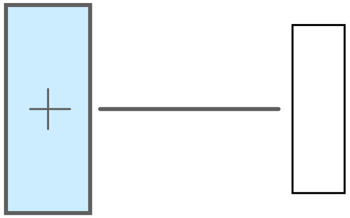
Let us look at the interface from 2D to 3D:



- The force vector is composed and resolved w.r.t. to the body system in order to formulate the balance equation
- There is no need to transform the torque since the torque-vector points in direction of the rotation axis.

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  f0 = {frame_a.fx, frame_a.fy, fz};
  frame_b.R.T*f0 + frame_b.f = zeros(3);
  frame_a.t + frame_b.t[3] = 0;

  Connections.root(frame_b.R);
end PlanarToMultiBody
```
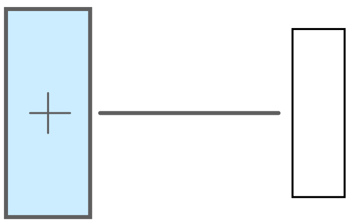

From 2D to 3D

Let us look at the interface from 2D to 3D:



- There remains a strange statement: “Connections.root(…)” It is needed for the handling of kinematic loops.

- We tell Dymola that this component is a potential source of overdetermination.

```
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end PlanarToMultiBody
```
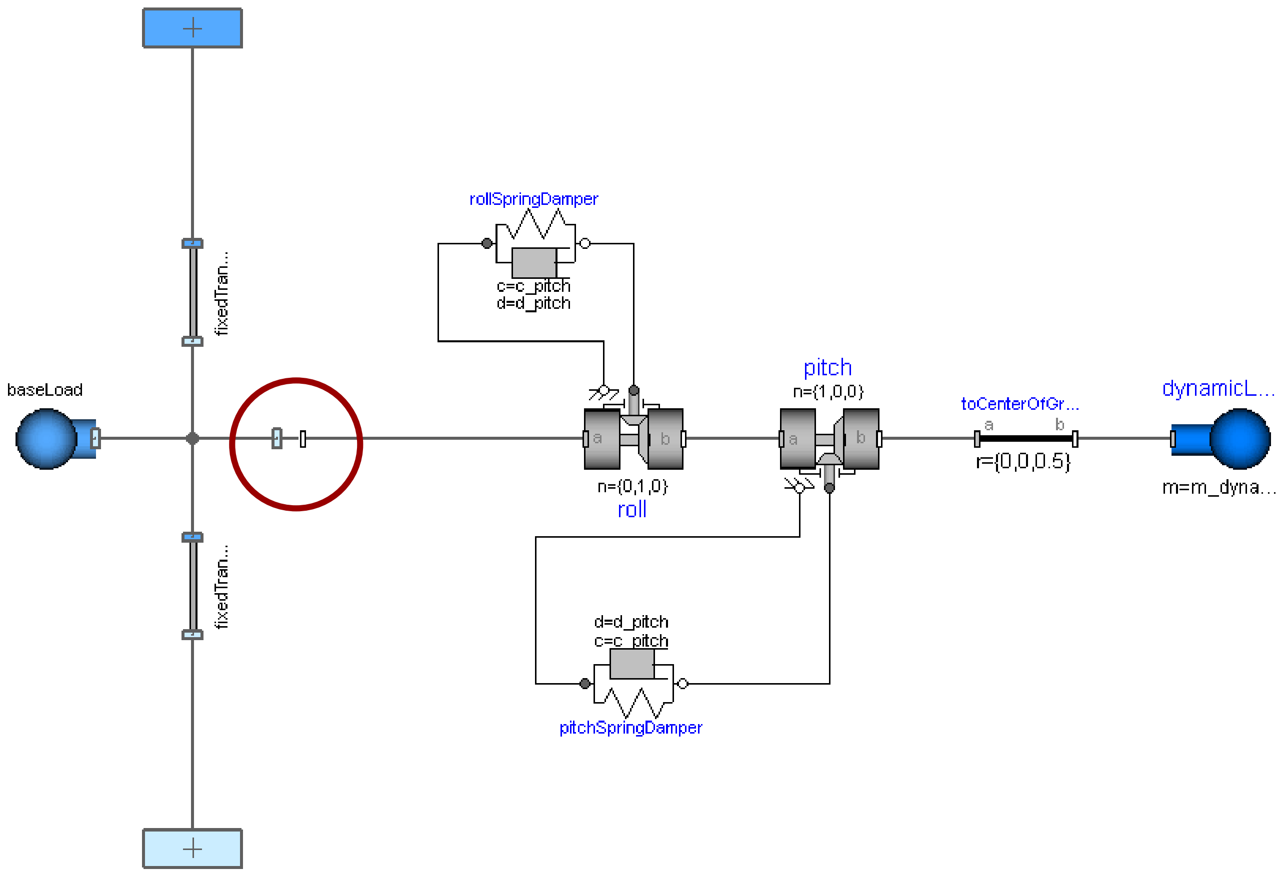

3D Chassis

Now, we apply this interface model to build a 3D chassis.



3D Chassis

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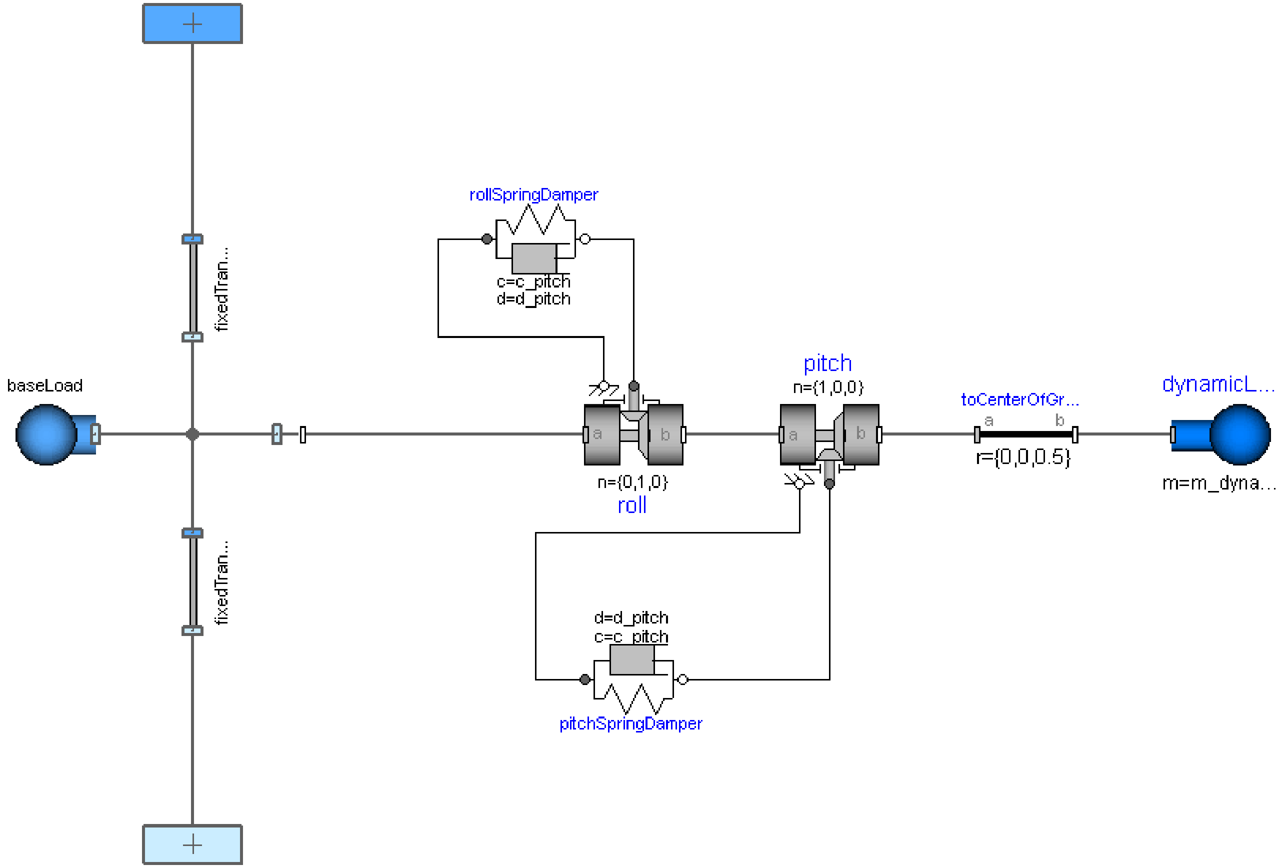
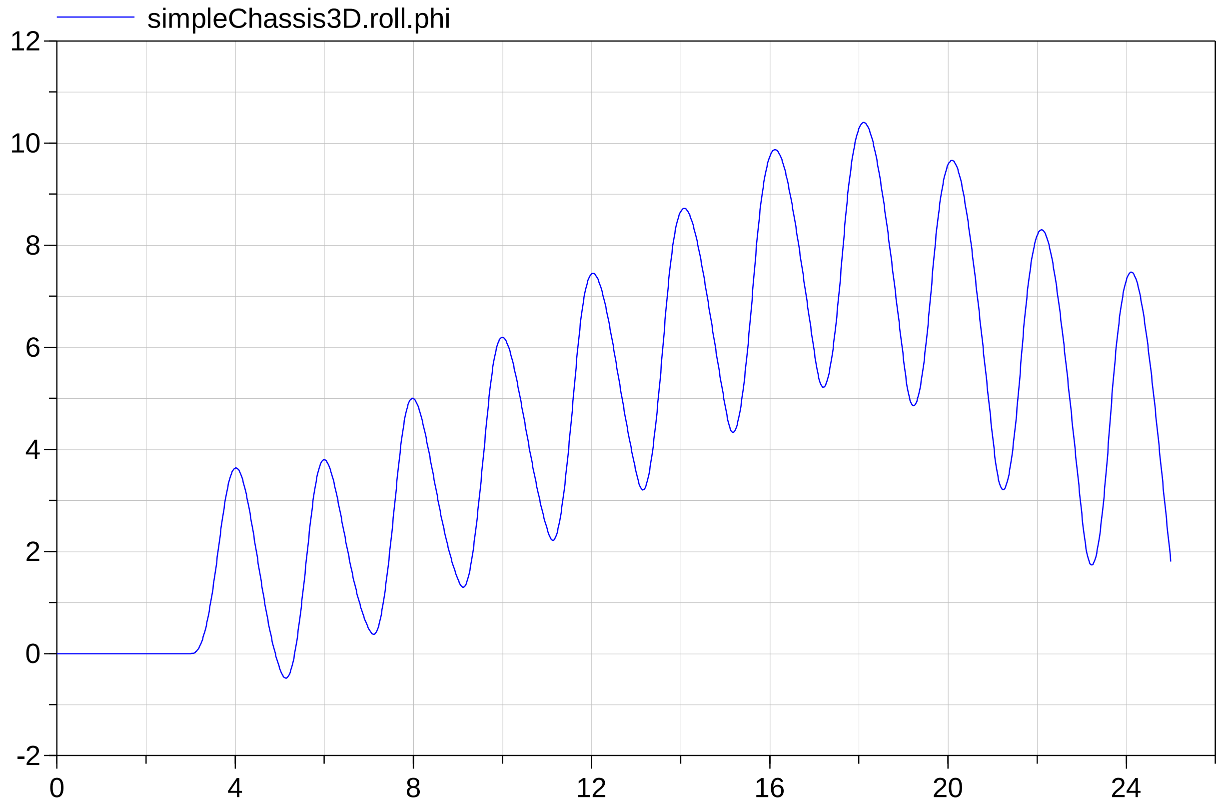
The dynamic load is represented by two revolute joints a mass and a rigid rod. Spring-Damper elements represent the flexibility of the suspension.



3D Chassis: Experiment

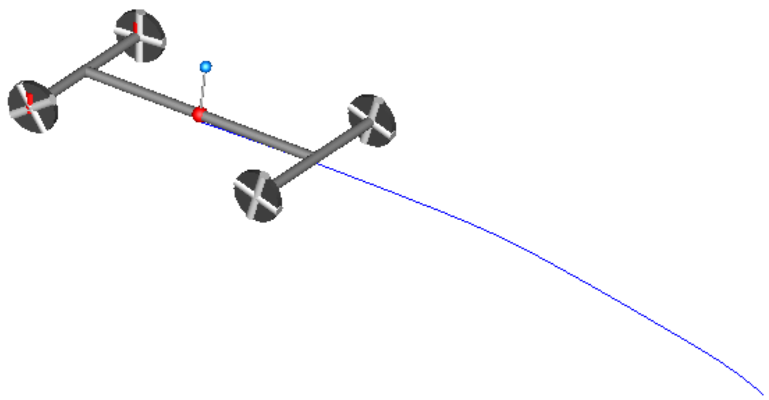
If we repeat the same experiment with the 3D-chassis we can observe the roll angle:



Summary

- The 3D chassis models the tilt of the chassis but not the dynamic influence on the normal load of the individual wheels.

- This will be your task in the last practical modeling exercise.



Questions ?